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# A comparative assessment of creep property predictions for a 1CrMoV rotor steel using the CRISPEN, CDM, Omega and Theta projection techniques

M. EVANS
*Materials Research Centre, School of Engineering, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP, UK*
*E-mail: m.evans@swansea.ac.uk*

This paper assesses the relative accuracy of a number of techniques that are capable of predicting a wide range of creep properties. The techniques studied in this paper include the 4-$\Theta$, 6-$\theta$, CRISPEN, Omega and CDM methodologies. The parameters of these models were estimated from short-term creep property data on 1CrMoV steel and these estimated models were then used to predict the known longer-term creep properties of this steel. It was found that the CDM approach yielded predicted times to failure that were wholly inadequate. The 6-$\theta$ approach was best for predicting long term times to failure. The best minimum creep rate predictions came from using either the 4-$\Theta$, or the CRISPEN or the Omega technique. Finally, times to small strains were best predicted using the 6-$\theta$ and CRISPEN techniques. 

## 1. Introduction

When designing materials for high temperature service the design criteria for long-term operation must guarantee that creep deformation should not cause excessive distortion over the planned service life and that creep failure should not occur within such a required operating life. Such creep fracture represents an obvious 'life limiting' design consideration as fracture of pipe work or other major components used by nuclear powered electricity generating plants could prove catastrophic. However, substantial problems can also be encountered due to excessive creep distortion. There are numerous examples of such deformation limits within the power generation and aero engine industries. For example, the blades of a steam turbine cannot be allowed to extend until they foul the surrounding casting. Similar requirements exist for the blades used in a gas turbine aero engine.

A number of projection techniques are well suited to the prediction of these two creep properties (time to failure and time to small strains). The aim of this paper is to assess the relative accuracy of such techniques in predicting these and other creep properties. The techniques to be compared in this paper include the 4-$\Theta$ and 6-$\theta$ projection techniques developed by Evans [1, 2] and then applied by amongst others Evans [3] and Evans [4], the continuum damage techniques (CDM) developed by Kachanov [5], Rabotnov [6] and latter generalised by Othman and Hayhurst [7], the $\Omega$ methodology for the life assessment of components recently proposed by Prager [8] and applied by Keeble [9] and the CReep Strain Prediction for ENgineering alloys (CRISPEN) approach to interpolation and extrapolation described by Ion *et al*. [10]. The traditional parametric procedures, such as the Larson-Miller technique [11], are not considered here because they are limited to the prediction of times to failure. In contrast, the approaches mentioned above allow the whole creep curve to be extrapolated to design (low) stresses from accelerated stresses. Time to any strain can then be 'read off' from such extrapolated creep curves.

The 1CrMoV rotor steel data set published by Evans *et al*. [12] will be used to study the accuracy of the predictions made of time to various low strains, time to failure and time to minimum creep rates (including the minimum creep rate itself) using the above projection techniques. To achieve this aim, this paper is therefore structured as follows. First, the experimental procedure and databases used are discussed. The following section then reviews the various creep property prediction techniques so that the mechanics of prediction in each case are clearly understood. Section 4 then discusses the theoretical underpinnings behind each technique so that key differences between them become clear. Section 5 discusses the procedures used to estimate the unknown parameters of each prediction technique. Then in Section 6 the accuracy of the long-term predictions made for the minimum creep rate, time to failure and time to various low strains using the above prediction techniques are assessed. A final section concludes.

## 2. Experimental procedures

The batch of material used for the present investigation represents the lower bound creep strength properties anticipated for 1CrMoV rotor steels. The chemical composition of this batch of material (in wt%) was determined as 0.27%C, 0.22%Si, 0.77%Mn, 0.008%S, 0.015%P, 0.97%Cr, 0.76%Ni, 0.85%Mo, 0.39%V, 0.125%Cu, 0.008%Al and 0.017%Sn. Following oil quenching from 1238 K and tempering at 973 K, the material had a tensile strength of 741 MPa, elongation of 17%, reduction in area of 55% and a 0.2% proof stress of 618 MPa.

Eighteen test pieces, with a gauge length of 25.4 mm and a diameter of 3.8 mm, were tested in tension over a range of stresses at 783, 823 and 863 K using high precision constant-stress machines [13]. At 783 K, six specimens were placed on test over the stress range 425 to 290 MPa, at 823 K seven specimens were placed on test over the stress range 335 to 230 MPa and at 863 K six specimens were tested over the stress range 250 to 165 MPa. Up to 400 creep strain/time readings were taken during each of these tests. Normal creep curves were observed under all these test conditions.

These eighteen specimens represent the accelerated test data from which the parameters of the above creep property prediction techniques will be estimated. To assess the extrapolative capability of these techniques long-term property data was supplied independently by an industrial consortium involving GEC-Alsthom, Babcocks Energy, National Power, PowerGen and Nuclear Electric. These long-term properties came from the same batch of material used in the accelerated test programme described above but for specimens with gauge lengths of 125 mm and diameters of 14 mm that were subjected to tests on high sensitivity constant-load tensile creep machines. As such it is to be expected that accurate failure time predictions made from the short-term constant stress data are likely to exceed these longer-term constant load failure times, because constant load specimens experience accelerating stresses.

It is important to note that in all cases below the creep property projection techniques did not make use of this long-term property data. The creep property projection techniques use only the accelerated test data to predict the properties of these longer-term test results.

## 3. Methods of creep life prediction

### 3.1. The 4-$\Theta$ technique

The 4-$\Theta$ technique describes the shape of any creep curve displaying normal primary and tertiary stages by using four theta parameters through the equation

$$\varepsilon_{\mathrm{i,h}} = \Theta_{1,\mathrm{h}}(1 - e^{-\Theta_{2,\mathrm{h}} t_{\mathrm{i,h}}}) + \Theta_{3,\mathrm{h}}(e^{\Theta_{4,\mathrm{h}} t_{\mathrm{i,h}}} - 1) \quad (1a)$$

where $\varepsilon_{\mathrm{i,h}}$ is the creep strain recorded at time $t_{\mathrm{i}}$ (with $N$ such recordings in total) and at test condition $h$. In Equation 1a, $\Theta_{1,\mathrm{h}}$ quantifies the total primary strain, $\Theta_{2,\mathrm{h}}$ the curvature of the creep curve during primary creep, $\Theta_{3,\mathrm{h}}$ scales the tertiary creep strain and $\Theta_{4,\mathrm{h}}$ measures the curvature of the creep curve during tertiary creep at test condition $h$.

The idea is then to test various specimens at accelerated stresses ($\tau_{\mathrm{h}}$) and temperatures ($T_{\mathrm{h}}$) and to estimate for each of the resulting experimental creep curves the $\Theta$ parameters in Equation 1a using a non-linear optimisation technique. Each $\Theta_{\mathrm{j,h}}$ ($j = 1$ to 4) is then related to the accelerated test conditions through a simple linear extrapolation function of the form

$$\ln(\Theta_{\mathrm{j,h}}) = a_{0,\mathrm{j}} + a_{1,\mathrm{j}}\tau_{\mathrm{h}} + a_{2,\mathrm{j}} T_{\mathrm{h}} + a_{3,\mathrm{j}}\tau_{\mathrm{h}} T_{\mathrm{h}} \quad (1b)$$

where $\tau_{\mathrm{h}}$ is the stress associated with test condition $h$ and $T_{\mathrm{h}}$ the temperature associated with test condition $h$ (with $h = 1$, M). $a_{0,\mathrm{j}}$ to $a_{3,\mathrm{j}}$ are parameters that can be estimated using the linear least squares procedure. Alternatively, weighted least squares can be used to reflect the fact that each $\Theta_{\mathrm{j,h}}$ value is only an estimate of its true value. The weights used must reflect the different uncertainties associated with each $\Theta_{\mathrm{j,h}}$ estimate. Each $\Theta_{\mathrm{j}}$ can then be extrapolated to lower stresses and temperatures by simply substituting in the required test conditions into Equation 1b. Let $\tilde{\Theta}_{\mathrm{j,h}}$ represents such extrapolated theta values. It is then possible to use these values to predict a variety of creep properties at close to the operating conditions for a designed material. A prediction of the minimum creep rate can be found by first predicting the time to minimum creep rate at test condition $h$

$$t_{\mathrm{M,h}} = \frac{1}{\tilde{\Theta}_{2,\mathrm{h}} + \tilde{\Theta}_{4,\mathrm{h}}} \ln \frac{\tilde{\Theta}_{1,\mathrm{h}}\tilde{\Theta}_{2,\mathrm{h}}^2}{\tilde{\Theta}_{3,\mathrm{h}}\tilde{\Theta}_{4,\mathrm{h}}^2} \quad (1c)$$

and from this the minimum creep rate itself

$$\dot{\varepsilon}_{\mathrm{M,h}} = \tilde{\Theta}_{1,\mathrm{h}}\tilde{\Theta}_{2,\mathrm{h}} e^{-\tilde{\Theta}_{2,\mathrm{h}} t_{\mathrm{M,h}}} + \tilde{\Theta}_{3,\mathrm{h}}\tilde{\Theta}_{4,\mathrm{h}} e^{\tilde{\Theta}_{4,\mathrm{h}} t_{\mathrm{M,h}}} \quad (1d)$$

where $\dot{\varepsilon}_{\mathrm{M,h}}$ is the minimum creep rate at test condition $h$ that occurs at time $t_{\mathrm{M,h}}$ at test condition $h$. Similarly, a prediction of the time to reach some specified creep strain at test condition $h$, $\varepsilon_{\mathrm{s,h}}$, can be obtained by solving numerically for time $t$ in the equation

$$\tilde{\theta}_{1,\mathrm{h}}(1 - e^{-\tilde{\theta}_{2,\mathrm{h}} t}) + \tilde{\theta}_{3,\mathrm{h}}(e^{\tilde{\theta}_{4,\mathrm{h}} t} - 1) - \varepsilon_{\mathrm{s,h}} = 0. \quad (1e)$$

As a special case of this, the failure time $t_{\mathrm{F}}$ can be predicted by solving Equation 1e when $\varepsilon_{\mathrm{s,h}}$ equals the rupture strain. Of course this requires the rupture strain to be extrapolated to the required conditions as well. This is typical done using Equation 1b with $\ln(\varepsilon_{\mathrm{F,h}})$ replacing $\ln(\Theta_{\mathrm{j,h}})$, where $\varepsilon_{\mathrm{F,h}}$ is the rupture strain observed at the accelerated test condition $h$.

### 3.2. The 6-$\theta$ technique

The 6-$\theta$ technique describes the shape of any creep curve by using six theta parameters through the equation

$$\varepsilon_{\mathrm{i,h}} = \theta_{1,\mathrm{h}}(1 - e^{-\theta_{2,\mathrm{h}} t_{\mathrm{i,h}}}) + \theta_{3,\mathrm{h}}(e^{\theta_{4,\mathrm{h}} t_{\mathrm{i,h}}} - 1) + \theta_{5,\mathrm{h}}(1 - e^{-\theta_{6,\mathrm{h}} t_{\mathrm{i,h}}}) \quad (2a)$$

$\theta_{5,h}$ and $\theta_{6,h}$ are two additional parameters required to improve the fit of the creep curve to the experimental data at test condition h over the early primary stage. Again $\theta_j$ ($j = 1$ to 6) is then related to the accelerated test conditions through the simple linear extrapolation function

$$\ln(\theta_{j,h}) = b_{0,j} + b_{1,j}\tau_h + b_{2,j}T_h + b_{3,j}\tau_h T_h \qquad (2b)$$

where $\tau_h$ is the stress associated with test condition $h$ and $T_h$ the temperature associated with test condition $h$ ($j = 1$ to $M$). $b_{0,j}$ to $b_{3,j}$ are parameters that can be estimated using either the linear or weighted linear least squares techniques. Using Equation 2a gives a 6-$\theta$ projection technique, where for example, the time to minimum creep rate can be predicted by substituting in the extrapolated $\theta$ values from Equation 2b, $\tilde{\theta}_{j,h}$, into

$$\frac{\tilde{\theta}_{1,h}\tilde{\theta}_{2,h}^2}{\tilde{\theta}_{3,h}\tilde{\theta}_{4,h}^2}e^{t[-\tilde{\theta}_{2,h}-\tilde{\theta}_{4,h}]} + \frac{\tilde{\theta}_{5,h}\tilde{\theta}_{6,h}^2}{\tilde{\theta}_{3,h}\tilde{\theta}_{4,h}^2}e^{t[-\tilde{\theta}_{6,h}-\tilde{\theta}_{4,h}]} - 1 = 0 \qquad (2c)$$

and solving numerically for $t$ ($= t_{M,h}$) at test condition $h$. From this the minimum creep rate itself can be predicted as

$$\dot{\varepsilon}_{M,h} = \tilde{\theta}_{1,h}\tilde{\theta}_{2,h}e^{-\tilde{\theta}_{2,h}t_{M,h}} + \tilde{\theta}_{3,h}\tilde{\theta}_{4,h}e^{\tilde{\theta}_{4,h}t_{M,h}} + \tilde{\theta}_{5,h}\tilde{\theta}_{6,h}e^{-\tilde{\theta}_{6,h}t_{M,h}} \qquad (2d)$$

Again a prediction of the time to reach some specified creep stain, $\varepsilon_{s,h}$, can be obtained by solving numerically for $t$ in the equation

$$\tilde{\theta}_{1,h}(1 - e^{-\tilde{\theta}_{2,h}t}) + \tilde{\theta}_{3,h}(e^{\tilde{\theta}_{4,h}t} - 1) + \tilde{\theta}_{5,h}(1 - e^{-\tilde{\theta}_{6,h}t}) - \varepsilon_{s,h} = 0. \qquad (2e)$$

The failure time $t_F$ can be predicted by solving Equation 2e when $\varepsilon_{s,h}$ equals the rupture strain. This will again require the rupture strain to be extrapolated to the required conditions using Equation 2b with $\ln(\varepsilon_{F,h})$ replacing $\ln(\theta_{j,h})$, where $\varepsilon_{F,h}$ is the rupture strain observed at the accelerated test conditions.

### 3.3. The CRISPEN technique

The CRISPEN (CReep Strain Prediction for ENgineering alloys) approach to interpolation and extrapolation describes the evolution of creep rate in terms of a rate equation containing internal variables. For tertiary processes governed by intergranular cavitations, which is assumed to be the damage mechanism relevant for the material used in this study, the creep curve is given by the equation

$$\varepsilon_{i,h} = -\frac{1}{\Phi_{4,h}}\ln\{1 - \Phi_{3,h}\Phi_{4,h}t_{i,h} - \Phi_{1,h}\Phi_{4,h}(1 - e^{\Phi_{2,h}t_{i,h}})\} \qquad (3a)$$

$\Phi_{1,h}$ to $\Phi_{4,h}$ are parameters governing the evolution of strain with time at test condition $h$. Again $\Phi_i$ ($j = 1$ to 4) are then related to the accelerated test conditions through the simple linear extrapolation function

$$\ln(\Phi_{j,h}) = c_{0,j} + c_{1,j}\tau_h + c_{2,j}T_h + c_{3,j}\tau_h T_h \qquad (3b)$$

where $\tau_h$ is the stress associated with test condition $h$ and $T_h$ the temperature associated with test condition $h$ ($j = 1$ to M). $c_{0,j}$ to $c_{3,j}$ are parameters that can be estimated using either linear or weighted linear least squares techniques. The time to minimum creep rate can be predicted by substituting in the parameters extrapolated from Equation 3b, $\tilde{\Phi}_{j,h}$, into

$$\frac{\tilde{V}_h\left[\tilde{\Phi}_{1,h}\tilde{\Phi}_{2,h}^2 e^{-\tilde{\Phi}_{2,h}t}\right] - \tilde{U}_h[-\tilde{\Phi}_{3,h}\tilde{\Phi}_{4,h} - \tilde{\Phi}_{1,h}\tilde{\Phi}_{2,h}\tilde{\Phi}_{4,h}e^{-\tilde{\Phi}_{2,h}t}]}{\tilde{V}_h^2} = 0 \qquad (3c)$$

where

$$\tilde{U}_h = [\tilde{\Phi}_{3,h} + \tilde{\Phi}_{1,h}\tilde{\Phi}_{2,h}(1 - e^{-\tilde{\Phi}_{2,h}t})],$$
$$\tilde{V}_h = [1 - \tilde{\Phi}_{3,h}\tilde{\Phi}_{4,h}t - \tilde{\Phi}_{1,h}\tilde{\Phi}_{4,h}(1 - e^{-\tilde{\Phi}_{2,h}t})],$$

and solving numerically for $t (= t_{M,h})$ at test condition $h$. From this the minimum creep rate itself can be predicted as

$$\dot{\varepsilon}_{M,h} = \frac{[\tilde{\Phi}_{3,h} + \tilde{\Phi}_{1,h}\tilde{\Phi}_{2,h}(1 - e^{-\tilde{\Phi}_{2,h}t_{M,h}})]}{[1 - \tilde{\Phi}_{3,h}\tilde{\Phi}_{4,h}t_{M,h} - \tilde{\Phi}_{1,h}\tilde{\Phi}_{4,h}(1 - e^{-\tilde{\Phi}2,_{h}t_{M,h}})]} \qquad (3d)$$

Again a prediction of the time to reach some specified creep stain, $\varepsilon_{s,h}$, can be obtained by solving numerically for $t$ in the equation

$$-\frac{1}{\tilde{\Phi}_{4,h}}\ln\{1 - \tilde{\Phi}_{3,h}\tilde{\Phi}_{4,h}t - \tilde{\Phi}_{1,h}\tilde{\Phi}_{4,h}(1 - e^{\tilde{\Phi}_{2,h}t})\} - \varepsilon_{s,h} = 0. \qquad (3e)$$

The failure time $t_F$ can be predicted by solving Equation 3e when $\varepsilon_{s,h}$ equals the rupture strain. This will again require the rupture strain to be extrapolated to the required conditions using Equation 3b with $\ln(\varepsilon_{F,h})$ replacing $\ln(\Phi_{j,h})$, where $\varepsilon_{F,h}$ is the rupture strain observed at the accelerated test condition $h$.

### 3.4. The Omega technique

Many petroleum companies in the USA have adopted the MPC $\Omega$ methodology for the life assessment of components used at their plants. Indeed, this method has now been recognised for inclusion in API RP579 on fitness for service. Recently, Prager [8] has formalised one version of this $\Omega$ approach where the creep curve expression takes the form

$$\varepsilon_{i,h} = -\frac{1}{\Omega_{1,h}}\ln[1 - \Omega_{2,h}(\Omega_{1,h})t_{i,h}] \qquad (4a)$$

where $\Omega_{1,h}$ and $\Omega_{2,h}$ are parameters governing the evolution of strain with time at test condition $h$. Unlike the three techniques above, this approach does not model the primary stages of creep as the model contains only two parameters. From Equation 4a the remaining life of a component at test condition $h$ is given by

$$t_{\mathrm{F,h}} - t_{\mathrm{i,h}} = \frac{1}{\Omega_{2,\mathrm{h}}\Omega_{1,\mathrm{h}}}[e^{-\Omega_{1,\mathrm{h}}\varepsilon_{\mathrm{i,h}}} - e^{-\Omega_{1,\mathrm{h}}\varepsilon_{\mathrm{F,h}}}]$$

or after taking logs

$$\ln[t_{\mathrm{F,h}} - t_{\mathrm{i,h}}] = \ln\left\{\frac{1}{\Omega_{2,h}\Omega_{1,h}}\right\} + \ln[e^{-\Omega_{1,\mathrm{h}}\varepsilon_{\mathrm{i,h}}} - e^{-\Omega_{1,\mathrm{h}}\varepsilon_{\mathrm{F,h}}}] \quad (4b)$$

Again these parameters are then related to the accelerated test conditions through the simple linear extrapolation function

$$\ln(\Omega_{\mathrm{j,h}}) = d_{0,\mathrm{j}} + d_{1,\mathrm{j}}\tau_{\mathrm{h}} + d_{2,\mathrm{j}}T_{\mathrm{h}} + d_{3,\mathrm{j}}\tau_{\mathrm{h}}T_{\mathrm{h}} \quad (4c)$$

$d_{0,\mathrm{j}}$ to $d_{3,\mathrm{j}}$ are constants that can be estimated using linear or weighted linear least squares procedures. Letting $\tilde{\Omega}_{\mathrm{j,h}}$ represents extrapolated Omega values from Equation 4c, it is then possible to use these values to predict a variety of creep properties at close to the operating conditions for a designed material. A prediction of the minimum creep rate at test condition $h$ is quite straightforward

$$\dot{\varepsilon}_{\mathrm{M,h}} = \tilde{\Omega}_{2,\mathrm{h}}. \quad (4d)$$

A prediction of the time to failure at test condition $h$ is given by

$$t_{\mathrm{F,h}} = \frac{1 - e^{-\tilde{\Omega}_{1,\mathrm{h}}\varepsilon_{\mathrm{F,h}}}}{\tilde{\Omega}_{1,\mathrm{h}}\tilde{\Omega}_{2,\mathrm{h}}}. \quad (4e)$$

Of course this requires the rupture strain to be extrapolated to the required conditions as well. This is typical done using Equation 4c with $\ln(\varepsilon_{\mathrm{F,h}})$ replacing $\ln(\Omega_{\mathrm{j,h}})$, where $\varepsilon_{\mathrm{F,h}}$ is the rupture strain observed at the accelerated test conditions. A prediction of the time to reach some specified creep strain at test condition $h$, $t_{\mathrm{s,h}}$, can be obtained from

$$t_{\mathrm{s,h}} = \frac{1 - e^{-\tilde{\Omega}_{1,\mathrm{h}}\varepsilon_{\mathrm{s,h}}}}{\tilde{\Omega}_{1,\mathrm{h}}\tilde{\Omega}_{2,\mathrm{h}}} \quad (4f)$$

where $\varepsilon_{\mathrm{s,h}}$ is the specified strain.

### 3.5. Some CDM techniques

Othman and Hayhurst [7] generalised Kachanov's [5] and Rabotnov's [6] approaches to damage accumulation so that the resulting creep curve has both primary and tertiary components. They model a normalised creep curve at test condition $h$ using

$$\frac{\varepsilon_{\mathrm{i,h}}}{\varepsilon_{\mathrm{F,h}}} = 1 - \left[1 - \left(\frac{t_{\mathrm{i,h}}}{t_{\mathrm{F,h}}}\right)^{\mathrm{v_h}+1}\right]^{\Delta_{\mathrm{h}}} \quad (5a)$$

where $\Delta_{\mathrm{h}}$ and $\upsilon_{\mathrm{h}}$ are model parameters that should, theoretically, be independent of stress and temperature. In reality, and as identified by both Othman and Hayhurst [7] and Evans and Wang [14], these parameters do vary with stress and to a lesser extent temperature. The parameters $\Delta_{\mathrm{h}}$ and $\upsilon_{\mathrm{h}}$ are estimated from Equation 5a using a non linear least squares procedures applied to all the creep curves obtained from a number of different accelerated test conditions. The above authors ignored this and simply averaged over all the parameter estimates. This paper will try and account for this test dependency by using the same functional form as used in the above techniques

$$\begin{aligned}\ln(\upsilon_{\mathrm{h}}) &= e_{0,1} + e_{1,1}\tau_{\mathrm{h}} + e_{2,1}T_{\mathrm{h}} + e_{3,1}\tau_{\mathrm{h}}T_{\mathrm{h}} \\ \ln(\Delta_{\mathrm{h}}) &= e_{0,2} + e_{1,2}\tau_{\mathrm{h}} + e_{2,2}T_{\mathrm{h}} + e_{3,2}\tau_{\mathrm{h}}T_{\mathrm{h}}\end{aligned} \quad (5b)$$

The above normalised creep curve implies a creep strain equation of the form

$$\varepsilon_{\mathrm{i,h}} = \frac{A\tau_{\mathrm{h}}^{\mathrm{n}}t_{\mathrm{F,h}}^{\mathrm{v_h}+1}}{\Delta_{\mathrm{h}}(\mathrm{v_h}+1)}\left[1 - \left(1 - \left(\frac{t_{\mathrm{i,h}}}{t_{\mathrm{F,h}}}\right)^{\mathrm{v_h}+1}\right)^{\Delta_{\mathrm{h}}}\right] \quad (5c)$$

where $A$ and $n$ are additional model parameters. The estimated values for $\Delta_{\mathrm{h}}$ and $\upsilon_{\mathrm{h}}$ obtained from Equation 5a at a particular test condition, together with the failure time at that condition, are inserted into Equation 5c and a non linear least squares procedure used to obtain a value for $A\tau_{\mathrm{h}}^{\mathrm{n}}$. When this is repeated over all accelerated test conditions the estimated values for $\ln(A\tau_{\mathrm{h}}^{\mathrm{n}})$ can be used in the regression

$$\ln\left(A\tau_{\mathrm{h}}^{\mathrm{n}}\right) = e_{0,3} + e_{1,3}\tau_{\mathrm{h}} + e_{2,3}T_{\mathrm{h}} + e_{3,3}\tau_{\mathrm{h}}T_{\mathrm{h}} \quad (5d)$$

with least squares being used to estimate the parameters $e_{0,3}$ to $e_{3,3}$. This equation can be used to predict values for $A$ and $n$ at various temperatures. All the above parameter estimates can then be used to obtain creep property predictions. These creep property predictions are rather unusual because properties such as time to a particular creep strain and the minimum creep rate can not be predicted until the time to failure, $t_{\mathrm{F}}$, is first predicted. In the Othman and Hayhurst model the time to failure at stress condition $\tau_{\mathrm{h}}$ is predicted from,

$$t_{\mathrm{F,h}} = \left[\frac{\upsilon_{\mathrm{h}}+1}{(\phi_{\mathrm{h}}+1)\left(B\tau_{\mathrm{h}}^{\mathrm{m}}\right)}\right]^{1/\mathrm{v_h}+1} \quad (5e)$$

where $\phi_{\mathrm{h}}$, $B$ and $m$ are additional model constants. An estimate for $\phi_{\mathrm{h}}$ is given by $1-(n/1-\Delta_{\mathrm{h}}))$, where $\upsilon_{\mathrm{h}}$ and $\Delta_{\mathrm{h}}$ are predicted in the way described above. To predict failure times beyond the accelerated stress conditions the term $B\tau_{\mathrm{h}}^{\mathrm{m}}$ must also be extrapolated. Equation 5e is first rearranged to yield values for $B\tau_{\mathrm{h}}^{\mathrm{m}}$ at each accelerated test condition using the estimated values for $\upsilon_{\mathrm{h}}$ and $\phi_{\mathrm{h}}$ and the known accelerated failure time $t_{\mathrm{F}}$. These calculated values for $\ln(B\tau_{\mathrm{h}}^{\mathrm{m}})$ can be used in the regression

$$\ln(B\tau_h^m) = e_{0,4} + e_{1,4}\tau_h + e_{2,4}T_h + e_{3,4}\tau_h T_h \quad (5f)$$

with least squares being used to estimate the parameters $e_{0,4}$ to $e_{3,4}$. This equation can be used to predict values for $B$ and $m$ at various temperatures. These predictions, once substituted into Equation 5e, give predictions for failure times at any test condition.

These Equations imply that at the time of failure the creep rate is infinite and the effective cross sectional area zero. This is never the observed case and if this assumption is relaxed the above equations generalise to

$$\frac{\varepsilon_{i,h}}{\varepsilon_{F,h}} = \frac{1 - \left[1 - \omega^*_{F,h}\left(\frac{t_{i,h}}{t_{F,h}}\right)^{v_h+1}\right]^{\Delta_h}}{1 - (1 - \omega^*_{F,h})^{\Delta_h}} \quad (5g)$$

$$\varepsilon_{i,h} = \frac{A\tau_h^n t_{F,h}^{v_h+1}}{\Delta_h(\phi_h + 1)\omega^*_{F,h}} \times \left[1 - \left(1 - \omega^*_{F,h}\left(\frac{t_{i,h}}{t_{F,h}}\right)^{v_h+1}\right)^{\Delta_h}\right] \quad (5h)$$

$$t_{F,h} = \left[\frac{(v_h + 1)[1 - (1 - \omega_{F,h})^{\phi+1}]}{(\phi_h + 1)(B\tau_h^m)}\right]^{1/v_h+1} \quad (5i)$$

where $\omega^*_{F,h}$ is an additional parameter related to damage accumulation as given by $\omega_{F,h}$.. This is explained in more detail in sub Section 4.4 below. This is an additional parameter that requires estimation in Equation 5g. Equations 5g to 5i can be used in the same way as Equations 5a to 5f to obtain creep property predictions with one addition. The value for $\omega^*_{F,h}$ can also be made a function of stress so that

$$\ln(\omega^*_{F,h}) = e_{0,5} + e_{1,5}\tau_h + e_{2,5}T_h + e_{3,5}\tau_h T_h \quad (5j)$$

where $e_{0,5}$ to $e_{3,5}$ are estimated by ordinary least squares.

## 4. Theoretical underpinnings of the above prediction techniques

### 4.1. The theta techniques

Evans [15] postulated the following constitutive equation for the creep rate

$$\dot{\varepsilon} = \dot{\varepsilon}_0(1 + H + R + W) \quad (6a)$$

where $\dot{\varepsilon}_0$ is the initial creep rate, $H$ is overall hardening, $R$ overall recovery and $W$ overall damage. These variables were in turn assumed to be governed by

$$\dot{H} = -\hat{H}\dot{\varepsilon} \quad (6b)$$

$$\dot{R} = \hat{R} \quad (6c)$$

$$\dot{W} = \hat{W}\dot{\varepsilon} \quad (6d)$$

where $\hat{H}$, $\hat{R}$ and $\hat{W}$ are positive quantities that are functions of stress and temperature. Integration of Equations 6 for conditions of constant stress and temperature gives

$$\varepsilon = \frac{1}{\hat{H}\dot{\varepsilon}_0}\left(\dot{\varepsilon}_0 - \frac{\hat{R}}{\hat{H}}\right)(1 - e^{-\hat{H}\dot{\varepsilon}_0 t}) + \frac{1}{\hat{W}}\left(e^{\frac{\hat{W}\hat{R}}{\hat{H}}t} - 1\right) \quad (7a)$$

Equation 7a has the same functional form as the 4-Θ creep curve given by Equation 1a with

$$\Theta_1 = \frac{1}{\hat{H}\dot{\varepsilon}_0}\left(\dot{\varepsilon}_0 - \frac{\hat{R}}{\hat{H}}\right); \quad \Theta_2 = \hat{H}\dot{\varepsilon}_0; \quad \Theta_3 = \frac{1}{\hat{W}};$$
$$\hat{\Theta}_4 = \frac{\hat{W}\hat{R}}{\hat{H}} \quad (7b)$$

and so each $\Theta_j$ is expected to be some function of stress and temperature.

Evans [2] and Evans [4] have shown that a better fit to the early primary part of the creep curve can be improved through the following generalisation of Equation 1a

$$\varepsilon_i = \eta(\theta) = \sum_{j=1}^{q}\Theta_{2j-1}(1 - e^{-\Theta_{2j}}) \quad (7c)$$

If $\Theta_{2j-1} > 0$, the jth term in this series represents a process which has a creep rate decreasing with increasing time (e.g., a normal primary curve). If $\Theta_{2j-1} < 0$ and $\Theta_{2j} < 0$ the term has a rate which increases with increasing time (e.g., a tertiary process). The fit of this model to any experimental creep curve can be made as close as desired by just increasing the value of $q$. Although there is no theoretical limit to the value of $q$, each term in Equation 7c needs to be capable of a theoretical explanation in terms of micro mechanisms governing high temperature creep. Also, estimating Equation 7c when $q$ is large presents huge practical problems in terms of being able to actually estimate all the $\Theta_j$ values. Primary and tertiary creep in precipitation hardened creep resisting alloys are known to be well represented by the first and second terms in Equation 7c so that agreement to experimental observation may be achieved by using Equation 2a. $\theta_5$ and $\theta_6$ are two additional parameters required to improve the fit of the creep curve to the experimental data over the early primary stage. As suggested in Evans [2], $\theta_5$ and $\theta_6$ may describe strains that are not permanent so that they may be a simple description of anelastic behaviour immediately after loading and therefore are recoverable on removal of the load.

### 4.2. The CRISPEN technique

The CRISPEN (CReep Strain Prediction for ENgineering alloys) approach to interpolation and extrapolation describes the evolution of creep rate in terms of a rate equation containing internal variables. This is supplemented by further differential equations which govern the evolution of the internal variables. Primary creep is modelled through a set of internal stress variables governed by work hardening and recovery. Tertiary creep is controlled by the growth of a damage parameter and various choices of growth mechanism are postulated. The method then seeks to construct creep curves under general conditions of stress and temperature by integrating together the coupled differential equations. Since a choice of mechanisms (which affect the shape of the creep curve) is offered, some knowledge of creep behaviour is required in advance.

For tertiary processes governed by intergranular cavitations, the differential equations are

$$\dot{\varepsilon} = \dot{\varepsilon}_0(1 - S)e^{\omega} \tag{8a}$$

$$\dot{S} = H\dot{\varepsilon}_0(1 - S) - RS \tag{8b}$$

$$\dot{\omega} = C\dot{\varepsilon}. \tag{8c}$$

where $S$ is an internal stress term and $\omega$ a damage term, both of which are zero at the start of creep. $H$, $R$ and $S$ are material constants representing hardening, recovery and damage rates. These later quantities must be obtained from the experimental creep curves. It is not clear how they should depend on applied stress and temperature. Thus, measured steady state creep rates, times in tertiary and primary creep and Norton's exponent are required and the dependence of these on conditions is not well known especially at the low stresses encountered in service. However, under conditions of constant stress and temperature, Equations 8 can be integrated in closed form to give Equation 3a with

$$\Phi_1 = \frac{H\dot{\varepsilon}_0^2}{(H\dot{\varepsilon}_0 + R)^2}, \quad \Phi_2 = H\dot{\varepsilon}_0 + R,$$

$$\Phi_3 = \frac{R\dot{\varepsilon}_0}{H\dot{\varepsilon}_0 + R} \quad \text{and} \quad \Phi_4 = C. \tag{8d}$$

As such the CRISPEN parameters, $\Phi_{\mathrm{j}}$ ($j = 1$ to 3 only), should also depend on stress and temperature.

## 4.3. The Omega technique

Recently, Prager [8] has formalised the above version of the $\Omega$ approach. It starts with the following differential equation

$$\dot{\varepsilon} = \frac{\mathrm{d}\varepsilon_t}{\mathrm{d}t} = \Omega_2\left[\frac{\sigma}{\sigma_0}\right]^{B_1}\left[\frac{1}{1-\omega}\right]^{B_2} \tag{9a}$$

where $\sigma_0$ is the initial stress in a constant load creep test and $\omega$ is a damage parameter measuring the loss of cross sectional area. Creep damage is therefore visualised as the formation of cavities. $B_1$ and $B_2$ are model parameters. For a constant load creep test $\sigma/\sigma_0 = \exp(\varepsilon)$, so that Equation 9a simplifies to

$$\dot{\varepsilon} = \frac{\mathrm{d}\varepsilon_t}{\mathrm{d}t} = \Omega_2\exp(B_1\varepsilon)\left[\frac{1}{1-\omega}\right]^{B_2} \tag{9b}$$

Unfortunately, this is not conventionally integrated. In order to derive a creep curve expression, Prager assumes that simple exponentials can be used to model creep rate acceleration due to increasing damage and micro structural changes not associated with damage

$$\dot{\varepsilon} = \frac{\mathrm{d}\varepsilon_t}{\mathrm{d}t} = \Omega_2\exp(B_1\varepsilon)\left[\frac{1}{\exp(-B_3\varepsilon)}\right]\left[\frac{1}{\exp(-B_4\varepsilon)}\right] \tag{9c}$$

where $B_1$ accounts for the rate increase due to cross section reduction (i.e., the stress increase in a constant load test), $B_3$ corresponds to micro structural damage and $B_4$ is used to account for other micro structural factors associated with stress change. Integration of Equation 9c gives the creep curve shown in Equation 4a with $\Omega_1 = B_1 + B_3 + B_4$.

## 4.4. Some CDM techniques

To model primary as well as tertiary creep, Othman and Hayhurst [7] proposed the following pair of differential equations to model the rate of strain and damage accumulation

$$\frac{\mathrm{d}\omega_{\mathrm{F}}}{\mathrm{d}t} = \frac{B\sigma^{\mathrm{m}}}{(1-\omega_{\mathrm{F}})^{\phi}}t^{\mathrm{v}} \tag{10a}$$

$$\dot{\varepsilon} = \frac{\mathrm{d}\varepsilon_t}{\mathrm{d}t} = A\tau^{\mathrm{n}}t^{\mathrm{v}} \tag{10b}$$

where $\omega_{\mathrm{F}}$ is a damage parameter defined as the effective loss in cross sectional area resulting from cavity formation as creep proceeds

$$\omega_{\mathrm{F}} = \left(1 - \frac{A}{A_0}\right)$$

where $A_0$ is the original cross sectional area of the test specimen, and $A$ the remaining effective area. $A$, $B$, $m$, $v$, $\phi$, and $n$ are model constant that are supposed to be independent of test conditions. From Equation 10a

$$(1-\omega_{\mathrm{F}})^{\phi}\mathrm{d}\omega_{\mathrm{F}} = B\tau_{\mathrm{t}}^{\mathrm{m}}\mathrm{d}t \tag{10c}$$

which upon integration over the range $\omega_{\mathrm{F}} = 0$ to $\omega_{\mathrm{F}} = 1$ with $\omega_{\mathrm{F}} = 0$ when $t = 0$ and $\omega_{\mathrm{F}} = 1$ when $t = t_{\mathrm{F}}$ gives Equation 5e above. Now the indefinite integral of Equation 10c assuming $\omega_{\mathrm{F}} = 0$ when $t = 0$ is

$$[1-\omega_{\mathrm{F}}]^{\phi+1} = 1 - \left[\frac{t}{t_{\mathrm{F}}}\right]^{\mathrm{v}+1}$$

Substituting this equation into Equation 10b gives

$$\dot{\varepsilon} = \frac{\mathrm{d}\varepsilon_t}{\mathrm{d}t} = A\tau_{\mathrm{o}}^{\mathrm{n}}\left[\left(1 - \frac{t}{t_{\mathrm{F}}}\right)^{\mathrm{v}+1}\right]^{\frac{-n}{\phi+1}} \tag{10d}$$

which upon integration gives Equation 5c with $\Delta = -n/\phi + 1$. Repeating the above steps, but now allowing $\omega = \omega_{\mathrm{F}} \leq 1$ when $t = t_{\mathrm{F}}$ yields Equations (5g to 5i above). In Equation 5g and 5h the parameter $\omega_{\mathrm{F}}^*$ is related to $\omega_{\mathrm{F}}$ through the expression $\omega_{\mathrm{F}}^* = 1 - [1 - \omega_{\mathrm{F}}]^{\phi+1}$. Provided $\omega_{\mathrm{F}}$ is less than 1 the creep rate will be finite at the point of failure and the effective cross sectional area positive.

## 5. Estimation

### 5.1. Estimating various creep curves

The strain values $\varepsilon_{i,h}$, given by Equations 1a, 2a, 3a, 4a, and 5a, can be thought of as fitted strains, derived at test condition $h$, using each of the five prediction techniques discussed above. These fitted strains will differ from the observed $N$ strains by the size of the experimental scatter, $\nu_{i,h,}$ at test condition $h$. Letting $k$ represent each of the above prediction techniques, (so that $k = 1$ corresponds to the 4-$\Theta$ techniques and $k = 5$ to the CDM approach),

$$\varepsilon_{1,i,h} = \Theta_{1,h}(1 - e^{-\Theta_{2,h}t_{i,h}}) + \Theta_{3,h}(e^{\Theta_{4,h}t_{i,h}} - 1) + \nu_{1,i,h} = f_1(t_{i,h}; \Theta_{j,h}) + \nu_{1,i,h} \tag{11a}$$

$$\varepsilon_{2,i,h} = \theta_{1,h}(1 - e^{-\theta_{2,h}t_{i,h}}) + \theta_{3,h}(e^{\theta_{4,h}t_{i,h}} - 1) + \theta_{5,h}(1 - e^{-\theta_{6,h}t_{i,h}}) + \nu_{2,i,h} = f_2(t_{i,h}; \theta_{j,h}) + \nu_{2,i,h} \tag{11b}$$

$$\varepsilon_{3,i,h} = -\frac{1}{\Phi_{4,h}} \ln\{1 - \Phi_{3,h}\Phi_{4,h}t_{i,h} - \Phi_{1,h}\Phi_{4,h}(1 - e^{\Phi_{2,h}t_{i,h}})\} + \nu_{3,i,h} = f_3(t_{i,h}; \Phi_{j,h}) + \nu_{3,i,h} \tag{11c}$$

$$\varepsilon_{4i,h} = -\frac{1}{\Omega_{1,h}} \ln[1 - \Omega_{2,h}(\Omega_{1,h})t_{i,h}] + \nu_{4,i,h} = f_4(t_{i,h}; \Omega_{j,h}) + \nu_{4,i,h} \tag{11d}$$

$$\frac{\varepsilon_{5,i,h}}{\varepsilon_{F,h}} = \frac{1 - \left[1 - \omega_{F,h}\left(\frac{t_{i,h}}{t_{F,h}}\right)^{\nu_h+1}\right]^{\Delta_h}}{1 - (1 - \omega_{F,h})^{\Delta_h}} + \nu_{5,i,h} = f_5(t_{i,h}; (\upsilon_h; \Delta_h, \omega_{f,h})) + \nu_{5,i,h} \tag{11e}$$

where $f_k(\ )$ is the fitted stain corresponding to prediction technique $k$ at test condition $h$ and in each case the fitted strains depend on an unknown parameter vector,$\Psi_{k,h}$, and time. Thus for each of the techniques above,

$$\Psi_{1,h} = \begin{bmatrix} \Theta_{1,h} \\ \Theta_{2,h} \\ \Theta_{3,h} \\ \Theta_{4,h} \end{bmatrix}; \quad \Psi_{2,h} = \begin{bmatrix} \theta_{1,h} \\ \theta_{2,h} \\ \theta_{3,h} \\ \theta_{4,h} \\ \theta_{5,h} \\ \theta_{6,h} \end{bmatrix}; \quad \Psi_{3,h} = \begin{bmatrix} \Phi_{1,h} \\ \Phi_{2,h} \\ \Phi_{3,h} \\ \Phi_{4,h} \end{bmatrix};$$

$$\Psi_{4,h} = \begin{bmatrix} \Omega_{1,h} \\ \Omega_{2,h} \end{bmatrix}; \quad \Psi_{5,h} = \begin{bmatrix} \upsilon_h \\ \Delta_h \\ \omega_{f,h} \end{bmatrix} \tag{12}$$

The unknown parameters in the vectors of Equation 12 can be estimated from an experimental creep curve obtained at test condition $h$ using a non-linear least squares estimator. This estimator chooses a value for $\Psi_{k,h}$ such that

$$F_{k,h}(\Psi_{k,h}) = \sum_{i=1}^{N} \nu_{k,i,h}^2 \tag{13}$$

are minimised, where $N$ is the number of strain—time readings making up the experimental curve at test condition $h$. Minimising such a sum of squared random errors is a standard problem in non-linear optimisation and the method of Gauss–Newton is often used. This technique linearises Equations 1a, 2a, 3a, 4a and 5a by using a linear Taylor series expansion around an initial value for the $\Psi_{k,h}$ parameter vector, $\Psi_{k,h}^0$

$$\varepsilon_{k,i,h} - f_k\left(t_{i,h}; \Psi_{k,h}^0\right) + \sum_{j=1}^{p} X_{k,i,j}^0 \Psi_{k,h,j}^0 = \sum_{j=1}^{p} X_{k,i,j}^0 \Psi_{k,h,j}^1 + \lambda_{k,i,h}^0 \tag{14}$$

where $\Psi_{k,h,j}^0$ can be though of as an initial guess as to the true value for the $j$th row of $\Psi_{k,h}$. The value for $p$ depends on the prediction technique being used, so that $p = 4$ for the 4-$\theta$ and CRISPEN methods, $p = 6$ for the 6-$\theta$ technique, $p = 3$ for the CDM method and $p = 2$ for the Omega method. $\Psi_{k,h,j}^1$ is an improved estimate for the $j$th row of $\Psi_{k,h}$. $f_k(t_{i,h}; \Psi_{k,h}^0)$ are the strains evaluated using the $k$th prediction technique with $\Psi_{k,h} = \Psi_{k,h}^0$ and $\nu_{k,i,h} = 0$. $X_{k,i,j}^0$ is the ith value for the partial derivative of $f_k(t_{i,h}, \Psi_{k,h}^0)$ with respect to $\Psi_{k,h,j}$ as evaluated using $\Psi_{k,h,j} = \Psi_{k,h,j}^0$,

$$X_{k,i,j}^0 = \partial f_k\left(t_{i,h}, \psi_{k,h}^0\right) / \partial \psi_{k,h,j}^0 \tag{15}$$

$\lambda_{k,i,h}^0$ contains both the true random error $\nu_{k,i,h,}$ and an approximation error due to the linear Taylor series expansion as evaluated at $\Psi_{k,h,j} = \Psi_{k,h,j}^0$.

Equation 14 is a linear equation and the ordinary least squares formula can be applied to it to yield estimates for $\Psi_{k,h,j}^1$. That is, values for $\Psi_{k,h,j}^1$ are chosen so as to minimise

$$\sum_{i=1}^{N} \left(\lambda_{k,i,h}^0\right)^2$$

An iterative procedure now takes place with $\Psi_{k,h,j}^1$ replacing $\Psi_{k,h,j}^0$ in Equation 14. Applying least squares again to Equation 14 will yield a further improved estimated for $\Psi_{k,h,j}$. This iterative process continues until the successive estimates for $\Psi_{k,h,j}$ converge. Upon convergence, Equation 13 will have been minimised. Let $\tilde{\Psi}_{k,h,j}$ stand for such estimates of $\Psi_{k,h,j}$.

### 5.2. Estimating extrapolation functions

Equations 1b, 2b, 3b, 4c and 5b are the extrapolation functions for each creep property prediction technique and can be estimated by ordinary least squares. The creep life predictions from such estimates are said to be unweighted. The ˆ symbol will be used to represent the resulting estimates of the parameters for the extrapolation function. So, for example, $\hat{a}_{0,j}$ to $\hat{a}_{3,j}$ are the unweighted estimates for $a_{0,j}$ to $a_{3,j}$ in Equation 1b.

However, because the values for $\Psi_{k,h,j}$ are estimates obtained from the above optimisation procedure, they can't really be treated as fixed values. Rather, they are random variables that have a mean value and a corresponding variance. It therefore makes sense to weight each value for $\Psi_{k,h,j}$ by their corresponding variances before applying the least squares procedure to Equations 1b, 2b, 3b, 4c and 5b. Clearly, if a particular $\Psi_{k,h,j}$ value is estimated with little certainty that data point should have only a minor impact in determining the parameters of Equations 1b, 2b, 3b, 4c and 5b. As $\ln(\Psi_{k,h,j})$ values are used in Equations 1b, 2b, 3b, 4c and 5b the variance of each $\ln(\Psi_{k,h,j})$ is a measure of such certainty. These variances are given by

$$\mathrm{Var}[\ln(\Psi_{k,h,j})] = \ln\left[1 + \left(\frac{\mathrm{Var}[\Psi_{k,h,j}]}{(\Psi_{k,h,j})^2}\right)\right] \quad (16a)$$

where $\mathrm{Var}[\Psi_{k,h,j}]$ is the variance associated with the estimate made for $\Psi_{k,h,j}$, i.e., $\tilde{\Psi}_{k,h,j}$. This paper uses the approaches taken by Evans [16] who has recently shown how to make robust estimates for $\mathrm{Var}[\Psi_{k,h,j}]$. That is, ones that allow for the $\nu_{k,i,h}$ to be auto correlated in very general ways.

Those $\Psi_{k,h,j}$ estimates with large variances should carry little weight in the determination of each of the parameter in Equations 1b, 2b, 3b, 4c and 5b. Hence it makes sense to give each $\Psi_{k,h,j}$ estimate a weight given by

$$\varpi_{k,h,j} = \frac{1}{\mathrm{Var}[\ln(\Psi_{k,h,j})]} \quad (16b)$$

Evans [1] proposed that each variable in Equations 1b be multiplied by $\varpi_{k,h,j}$ and the ordinary least squares estimation technique applied to this weighted extrapolation function in order to obtain the required parameters. For, example, for the 4-$\Theta$ extrapolation function given by Equation 1b, the weighted least squares regression is

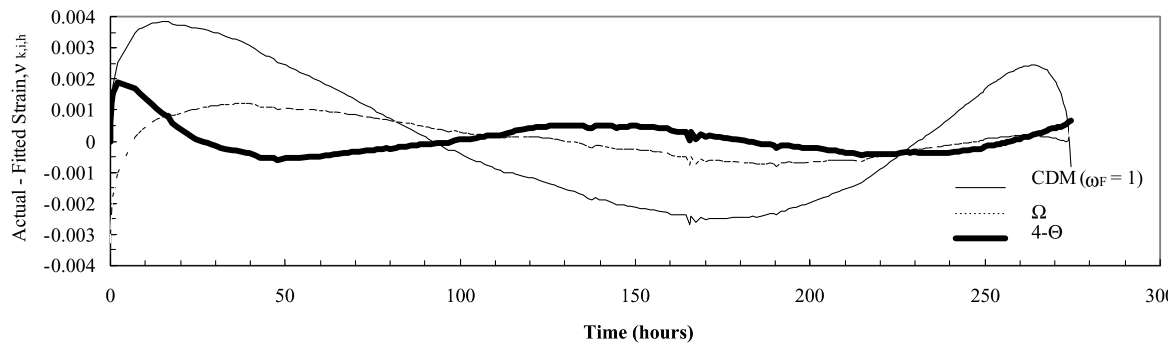
$$[\ln \theta_{j,h}]^* = b_{0,j} + b_{1,j}\tau_h^* + b_{2,j}T_h^* + b_{3,j}[\tau_h T_h]^* \quad (16c)$$

where $[\ln \theta_{j,h}]^* = \varpi_{1,h,j}[\ln \theta_{j,h}]$, $\tau_i^* = \varpi_{1,h,j}[\tau_h]$, $T_h^* = \varpi_{1,h,j}[T_h]$, $[\tau_r T_h]^* = \varpi_{1,h,j}[\tau_h T_h]$. The $\sim$ symbol will be used to represent the resulting estimates made of the parameters for the extrapolation function. So, for example, $\tilde{a}_{0j}$ to $\tilde{a}_{3j}$ are the weighted estimates for $a_{0j}$ to $a_{3,j}$ in Equation 1b.

## 6. Results

### 6.1. Fitted creep curves

Fig. 1 shows the difference between the actual creep strain recorded at various times at a stress of 270 MPa and a temperature of 823 K and the fitted strains given by Equations 1a, 2a, 3a, 4a and 5a. These are the $\nu_{k,i,h}$ values of Equation 11a to Equation 11e, with $\nu_{k,i,h}$ (in absolute terms) being large when the creep curve is a poor fit to the experimental data. The results shown are typical of those obtained at the other accelerated test conditions. A number of observations can be made about the techniques used to obtain the fitted creep curves. First, when using the CDM methodology, the fitted creep strains are very poor at all times if failure is forced to occur when $\omega_F = 1$. However, the fitted strains are very good (at all times) if $\omega_F$ is allowed to be different from unity. Indeed, under such circumstance the fitted strains are as good if not better than



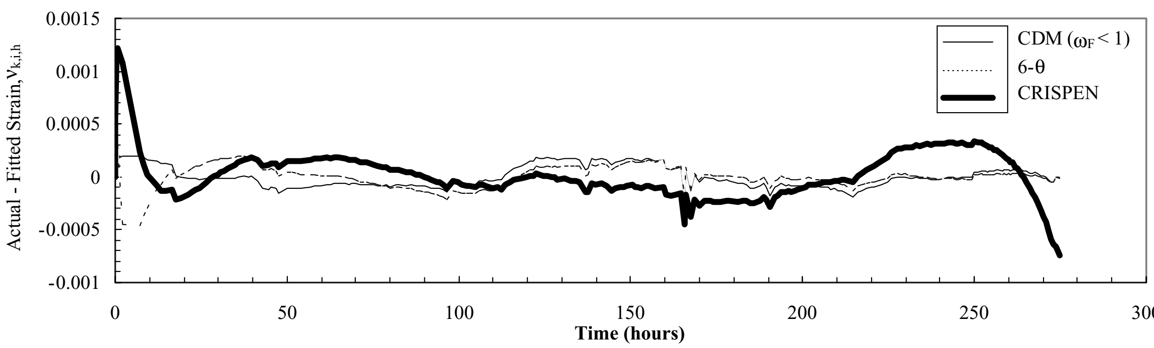



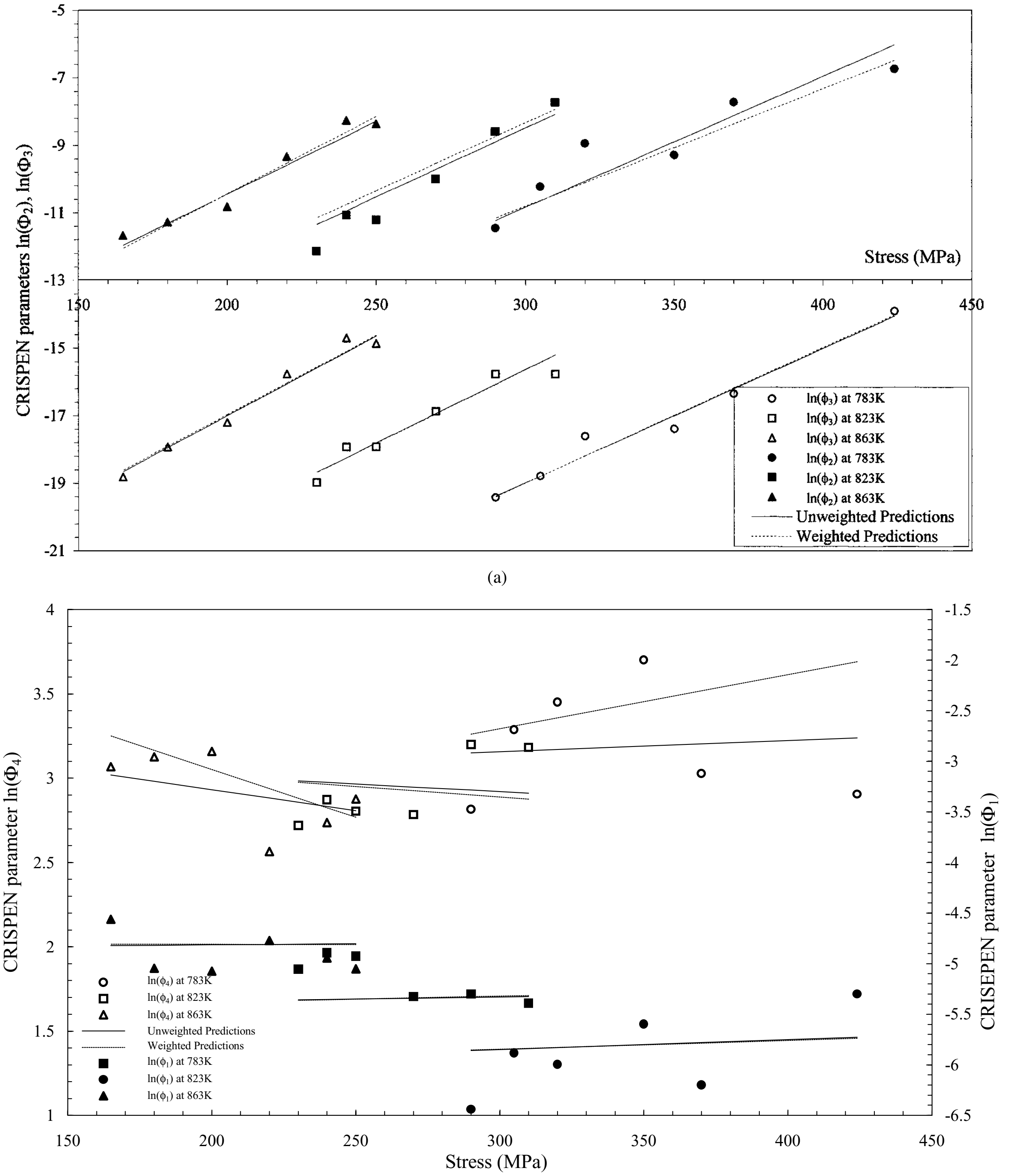

*Figure 1* Difference between experimental and fitted creep strains obtained using various prediction techniques at 823 K and 270 MPa.

those obtained from the other techniques. However, a technique that provides good fitting creep curves to the accelerated test data does not necessarily produce good extrapolated creep curves. What is also important is how well the parameters of the creep curve vary with test conditions and this will be looked at in the section below.

Secondly, the 6-$\theta$ technique gives a better fitted strain than those obtained from the 4-$\Theta$ technique—especially at low strains or times. Thirdly, the CRISPEN technique produces strain predictions that are better than those obtained using the 4-$\Theta$ technique and similar to those obtained using the 6-$\theta$ technique—except at small times where the 6-$\theta$ technique is far superior. Fourthly, the $\Omega$ technique yields fairly poor fitted strains that are very similar, for times in excess of 100 h, to those obtained using the CDM approach with $\omega_F = 1$. Finally, for all the techniques, the values for $\nu_{k,i,h}$ exhibit wave like behaviour and so are clearly autocorrelated. It is therefore important to use the robust estimation techniques described by Evans [16] to estimate $Var[\Psi_{k,h,j}]$ and thus the weights required for a weighted least squares estimate of the extrapolation functions.

## 6.2. Variation of $\Psi_{k,h,j}$ with test conditions

Figs. 2a and b show the estimated CRISPEN parameters as a function of the accelerated stress and temperature




(b)

*Figure 2* (a) The variation of $\Phi_2$ and $\Phi_3$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel. (b) The variation of $\Phi_1$ and $\Phi_4$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel.

test conditions. Also shown are the predicted values given by Equation 3b with $c_{0,\mathrm{j}}$ to $c_{3,\mathrm{j}}$ estimated using ordinary least squares (solid lines) and weighted least squares (dashed lines). It is clear that the parameters $\ln(\Phi_2)$ and $\ln(\Phi_3)$ show a strong dependency on test conditions with Equation 3b giving a very good fit to the data. This is in agreement with Equations 8d where $\Phi_2$ and $\Phi_3$ are shown to be functions of stress, given that the initial creep rate, $\dot{\varepsilon}_0$, is dependant upon stress.

However, $\ln(\Phi_1)$ and $\ln(\Phi_4)$ show less variation with stress and, in the case of $\ln(\Phi_4)$, substantially more scatter around the fitted trend lines. This is broadly in agreement with Equations 8d where $\Phi_4$ was shown to be a constant. Using weighted least squares alters only slightly the predicted values for $\Phi_1$, $\Phi_2$ and $\Phi_3$, but for $\Phi_4$ weighting has a significant impact on the predicted values for $\Phi_4$ (reflecting the large scatter present in the estimated $\Phi_4$ values).

Similar conclusions can be drawn from Figs 3 and 4 that show the 4-$\Theta$ and 6-$\theta$ parameters respectively as a function of the accelerated stress and temperature test conditions. Also shown are the predicted values given by Equations 1b and 2b with $a_{0,\mathrm{j}}$ to $b_{3,\mathrm{j}}$ estimated using ordinary least squares (solid lines) and weighted least

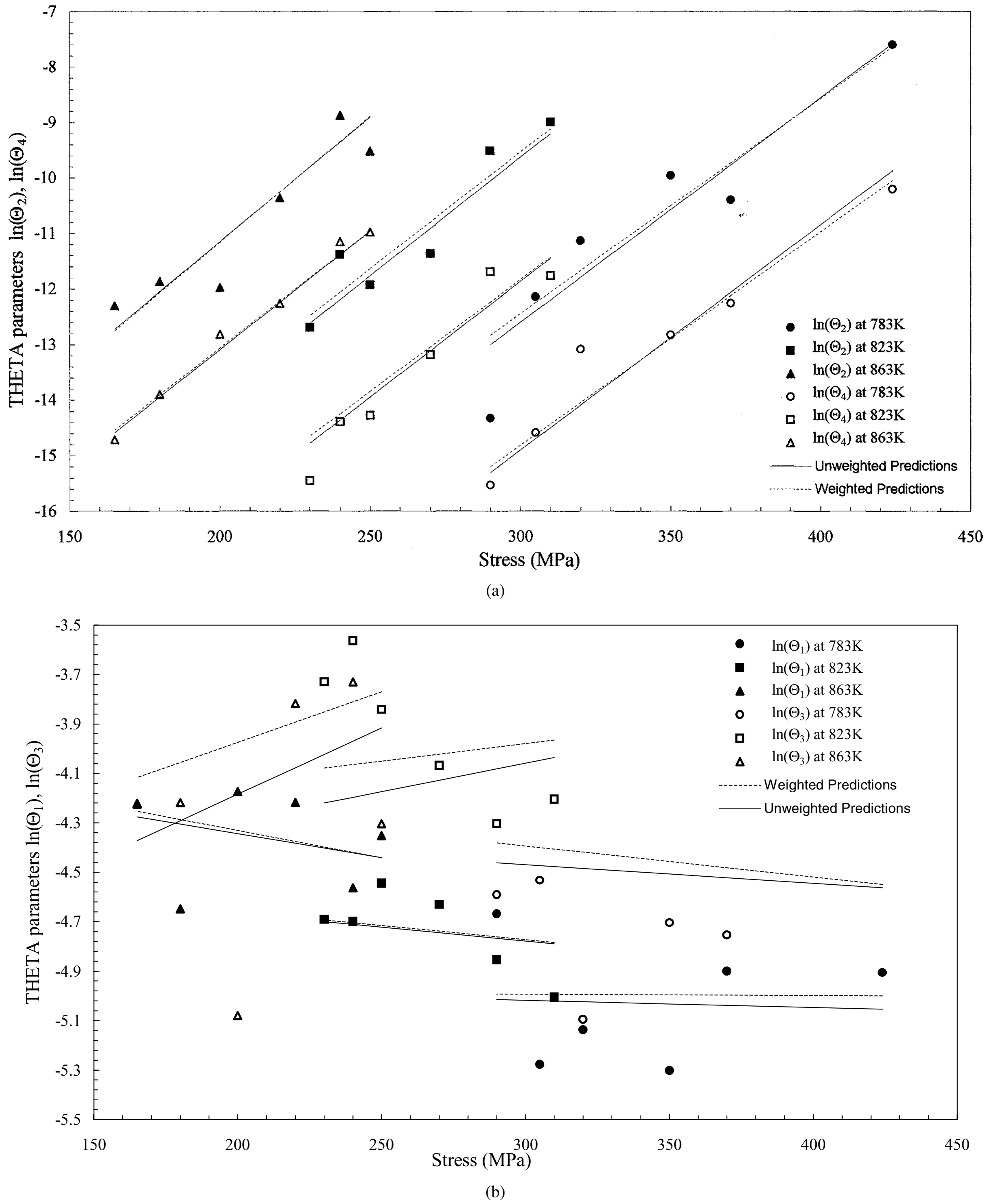
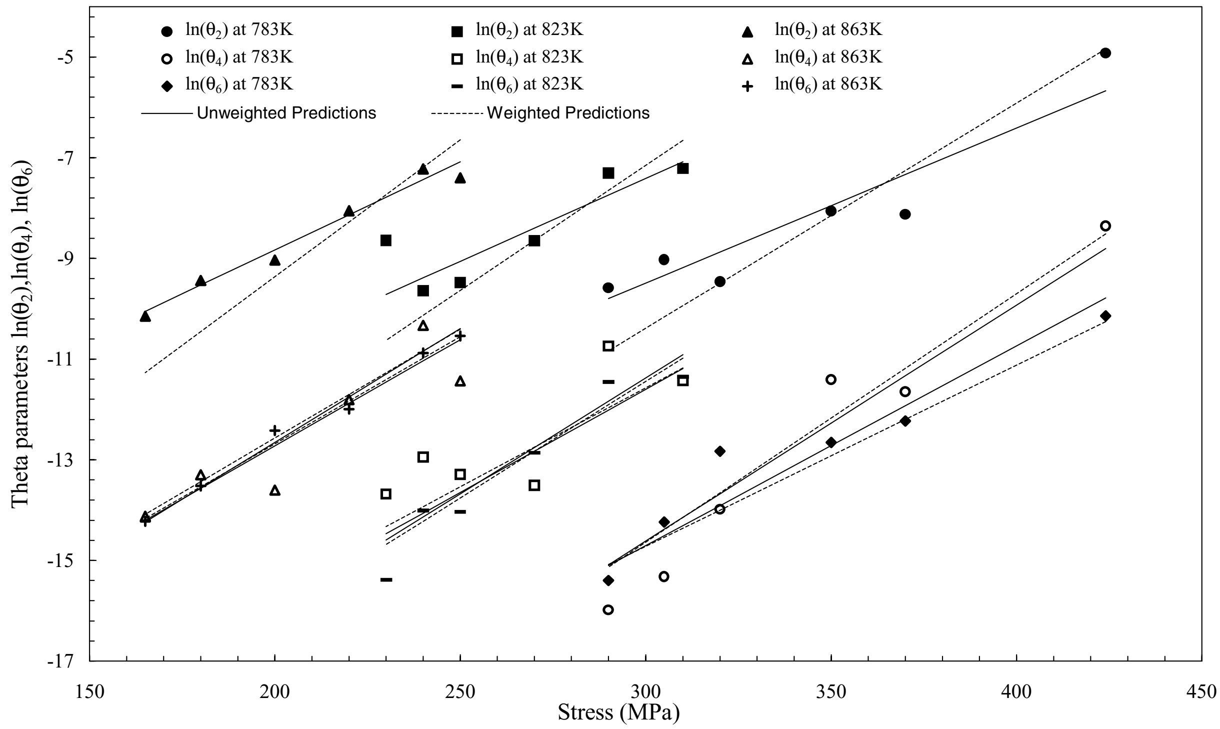
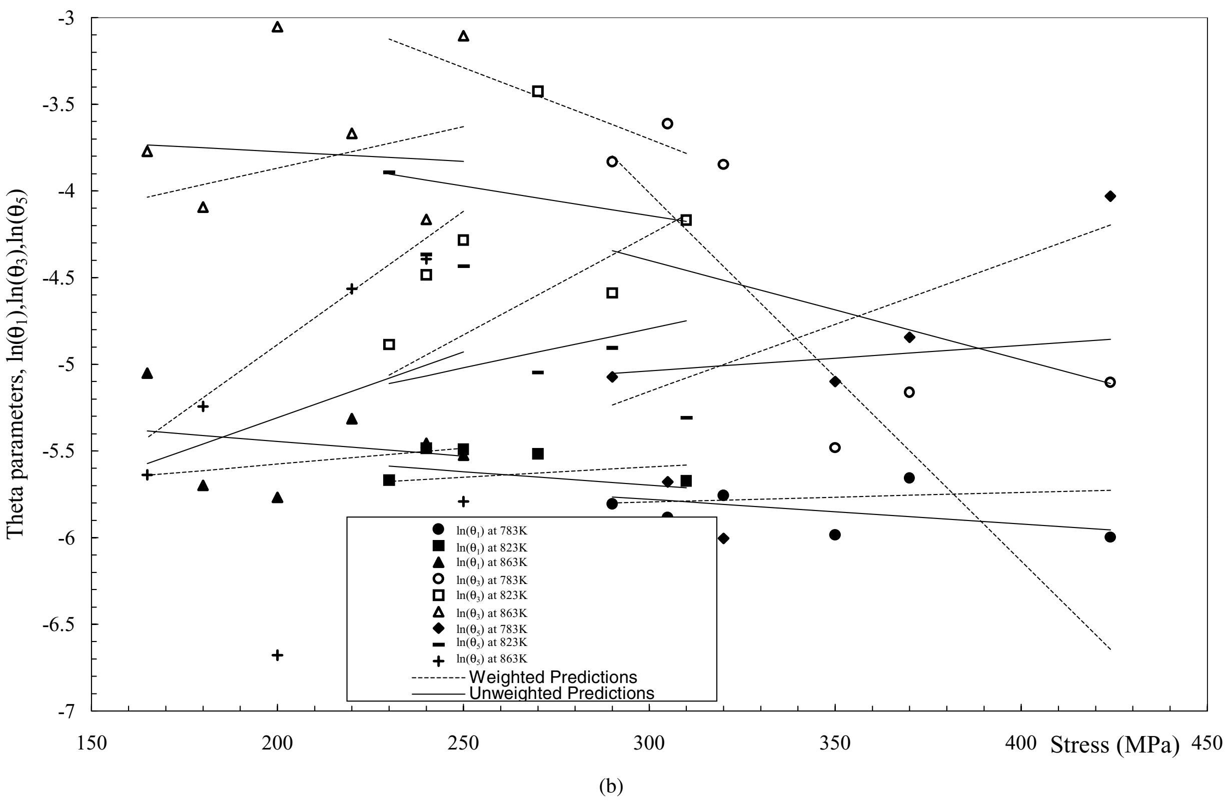



*Figure 3* (a) The variation of $\Theta_2$ and $\Theta_4$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel. (b) The variation of $\Theta_1$ and $\Theta_3$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel.




(a)

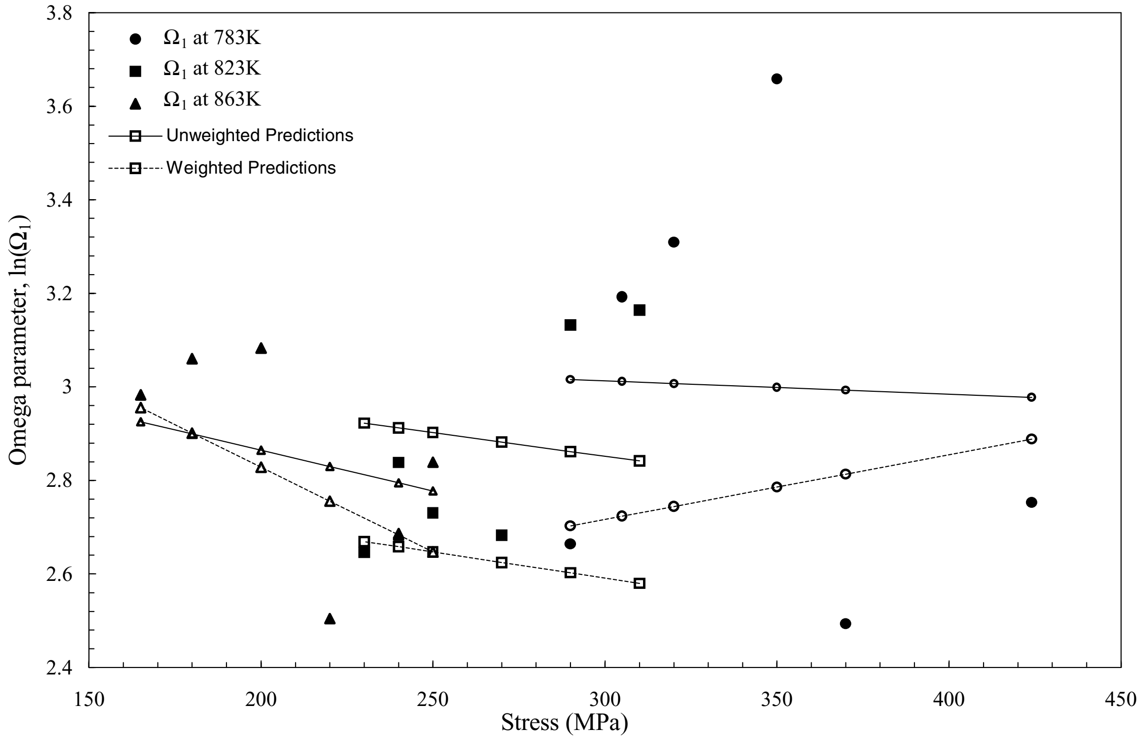



(b)

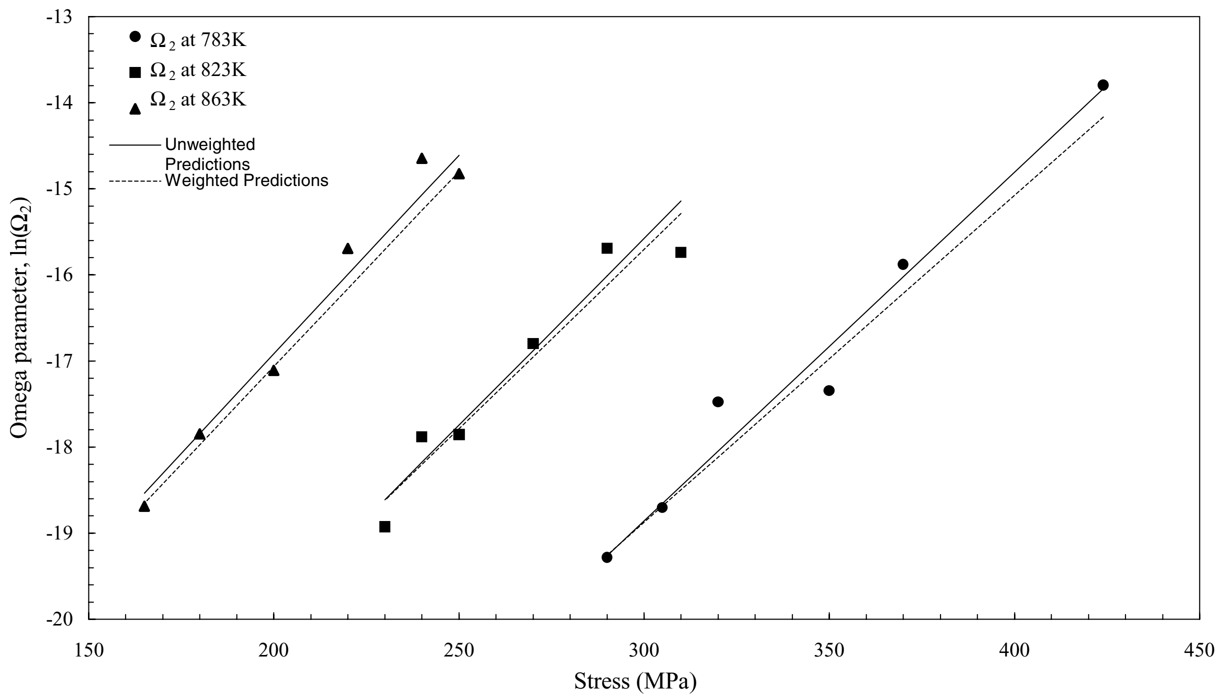
*Figure 4* (a) The variation of $\theta_2$, $\theta_4$ and $\theta_6$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel. (b) The variation of $\theta_1$, $\theta_3$ and $\theta_5$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel.

squares (dashed lines). It is clear that the parameters $\ln(\Theta_2)$, $\ln(\Theta_4)$, $\ln(\theta_2)$, $\ln(\theta_4)$ and $\ln(\theta_6)$ show a strong dependency on test conditions with Equations 1b and 2b giving a very good fit to the data. This is in agreement with Equations 6 and 7 where all the theta parameters are shown to be functions of stress. However, $\ln(\Theta_1)$, $\ln(\Theta_3)$, $\ln(\theta_1)$ $\ln(\theta_3)$ and $\ln(\theta_5)$ show less variation with stress with substantially more scatter around the fitted trend lines. Using weighted least squares alters only slightly the predicted values for $\ln(\Theta_2)$, $\ln(\Theta_4)$, $\ln(\theta_4)$ and $\ln(\theta_6)$, but for the remaining theta parameters weighting has a significant impact on the predicted values.

Fig. 5 show the estimated Omega parameters as a function of the accelerated stress and temperature test conditions. Also shown are the predicted values




(a)

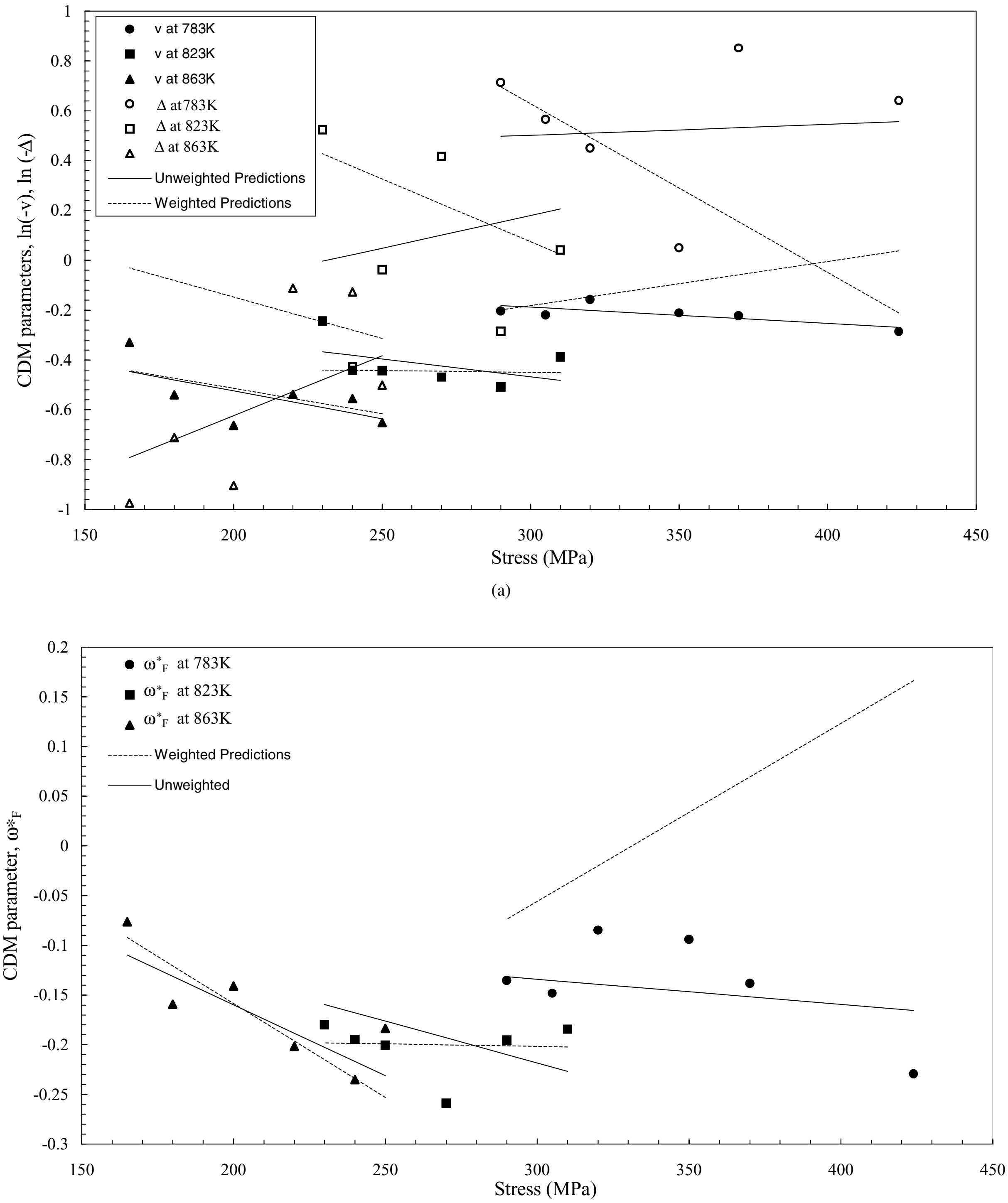



(b)

*Figure 5* (a) The variation of $\Omega_1$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel. (b) The variation of $\Omega_2$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel.

given by Equation 4c with $d_{0,\mathrm{j}}$ to $d_{3,\mathrm{j}}$ estimated using ordinary least squares (solid lines) and weighted least squares (dashed lines). It is clear that the parameter $\ln(\Omega_2)$ shows a strong dependency on test conditions with Equation 4c giving a very good fit to the data. Using weighted least squares alters only slightly the predicted values for $\ln(\Omega_2)$. However, $\ln(\Omega_1)$ shows less variation with stress with substantially more scatter around the fitted trend lines. As expected with such variation, weighting makes a big difference to the predicted values for $\ln(\Omega_2)$. These result are different to that of Prager [8] and Keeble [9] who found that it was $\ln(\Omega_1)$, rather than $\ln(\Omega_2)$, that varied most systematically with stress and temperature.

Fig. 6 show the estimated CDM parameters as a function of the accelerated stress and temperature test conditions. Also shown are the best fit lines estimated using ordinary least squares (solid lines) and weighted least squares (dashed lines). According to the CDM



(a)


(b)

*Figure 6* (a) The variation of v and Δ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel. (b) The variation of $\omega_F^*$ with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel.

model the parameters v and Δ in Equation 5a should strictly be independent of stress. Whilst the estimated values for these parameters show substantial random variation, they do also seem to vary to some extent with stress. However, this large variation may explain the poor extrapolated creep property predictions that stem from the above CDM model. The estimated dependency with stress is further seen to be heavily dependent upon whether ordinary or weighted least squares is used. Further, Fig. 6b shows that the value for $\omega_F^*$ at failure is also dependant upon stress, with this stress dependency being more well defined at the higher temperatures. Weighting has its greatest impact at the lowest temperature.

Fig. 7 shows that, whilst the rupture strain is dependent upon test conditions, this relationship is quite weak and subject to substantial variation. Whilst this may affect the predicted times to failure given by the above procedures (excluding CDM), it needs to be remembered that at the point of failure the creep curve is quite steep so that large variations in rupture strain are consistent with very similar times to rupture.

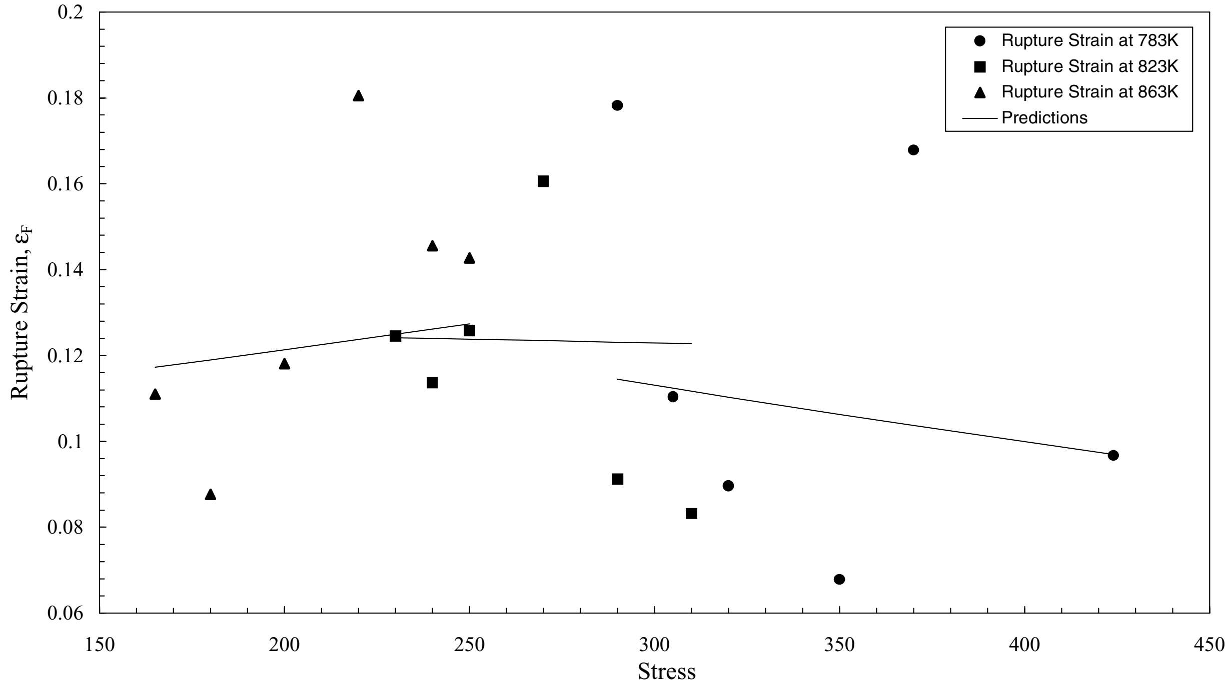
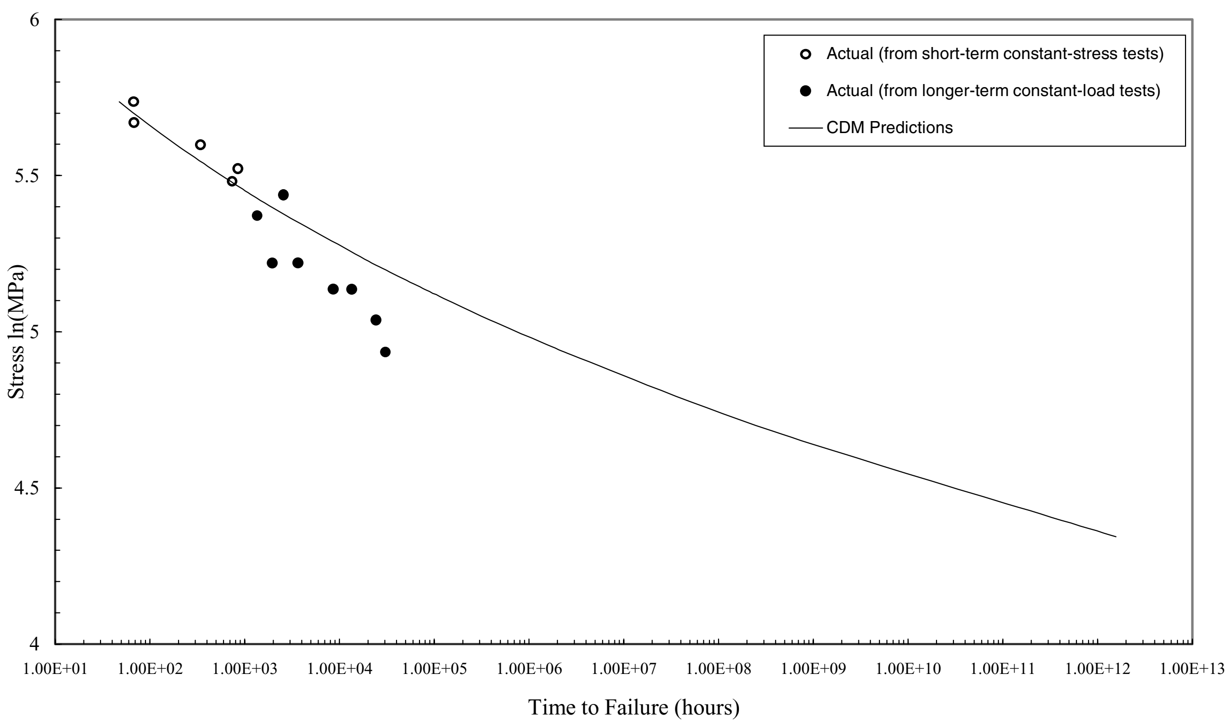



*Figure 7* The various of rupture strain with stress at 783, 823 and 863 K for 1Cr-1Mo-IV steel.

## 6.3. Creep property predictions

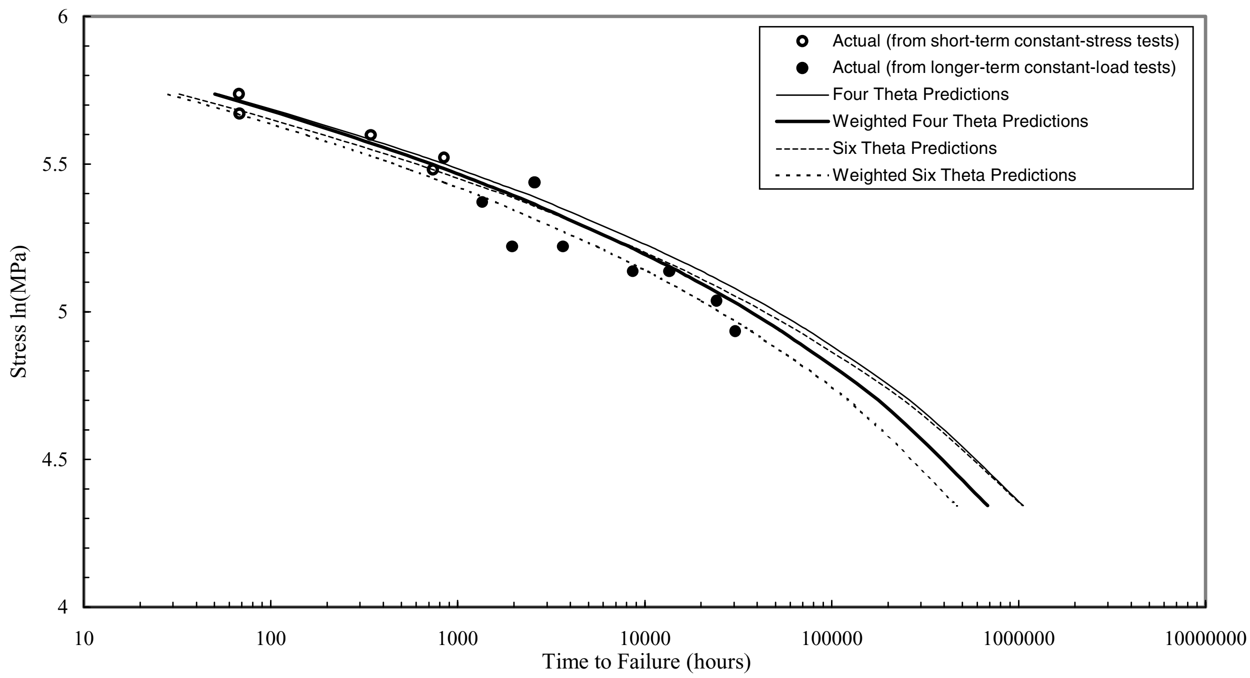
### *6.3.1. Times to failure*

Fig. 8a shows the failure time predictions obtained using Equations 5, with $\omega_F \leq 1$, together with the parameter estimates shown in Fig. 6. All the CDM parameters were estimated using the short-term constant stress data and not surprisingly the resulting predictions for these short times are very good. However, the longer-term predictions are not very good, especially at the lower stresses. As all the other creep

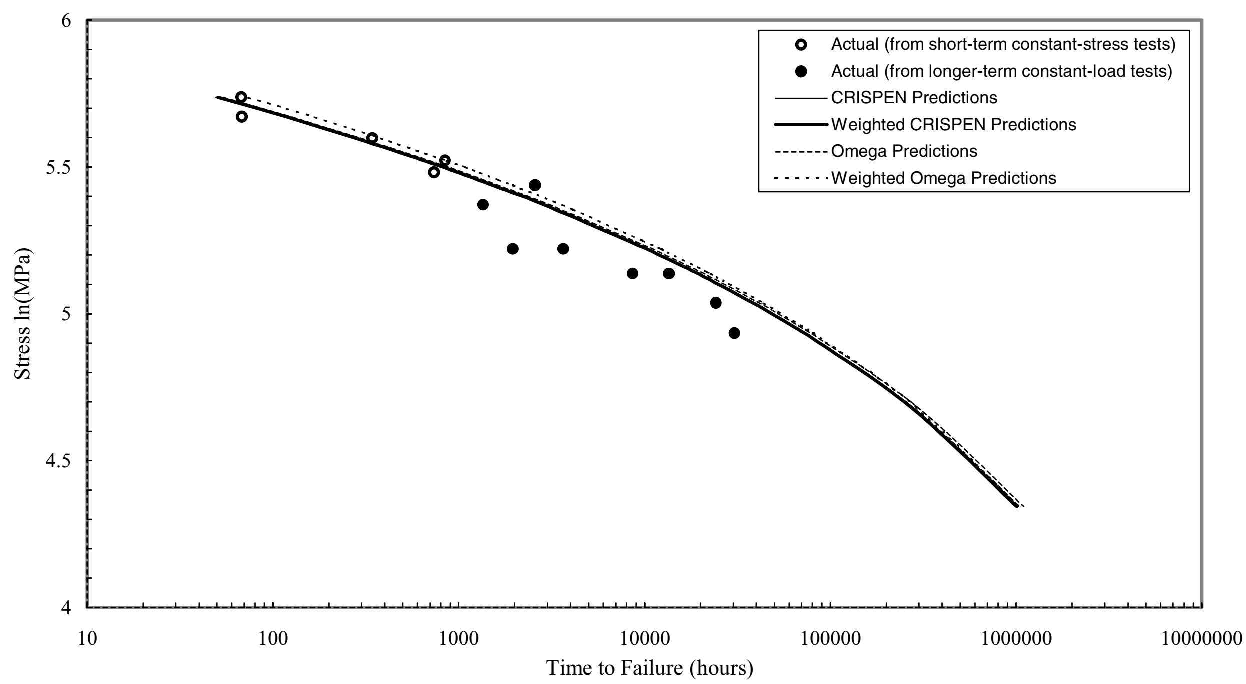



(a)

*Figure 8* (a) Predicted $\ln \tau / \ln t_F$ relationships obtained from CDM lifeing procedures. The plot includes the measured $t_F$ values from the short-term constant-stress tests at 823 K and the longer-term results from constant load tests at 823 K. (b) Predicted $\ln \tau / \ln t_F$ relationships obtained from various lifeing procedures. The plot includes the measured $t_F$ values from the short-term constant-stress tests at 823 K and the longer-term results from constant load tests at 823 K. (c) Predicted $\ln \tau / \ln t_F$ relationships obtained from various lifeing procedures. The plot includes the measured $t_F$ values from the short-term constant-stress at 823 K and the longer-term results from the constant load tests at 823 K. (*Continued.*)




(b)

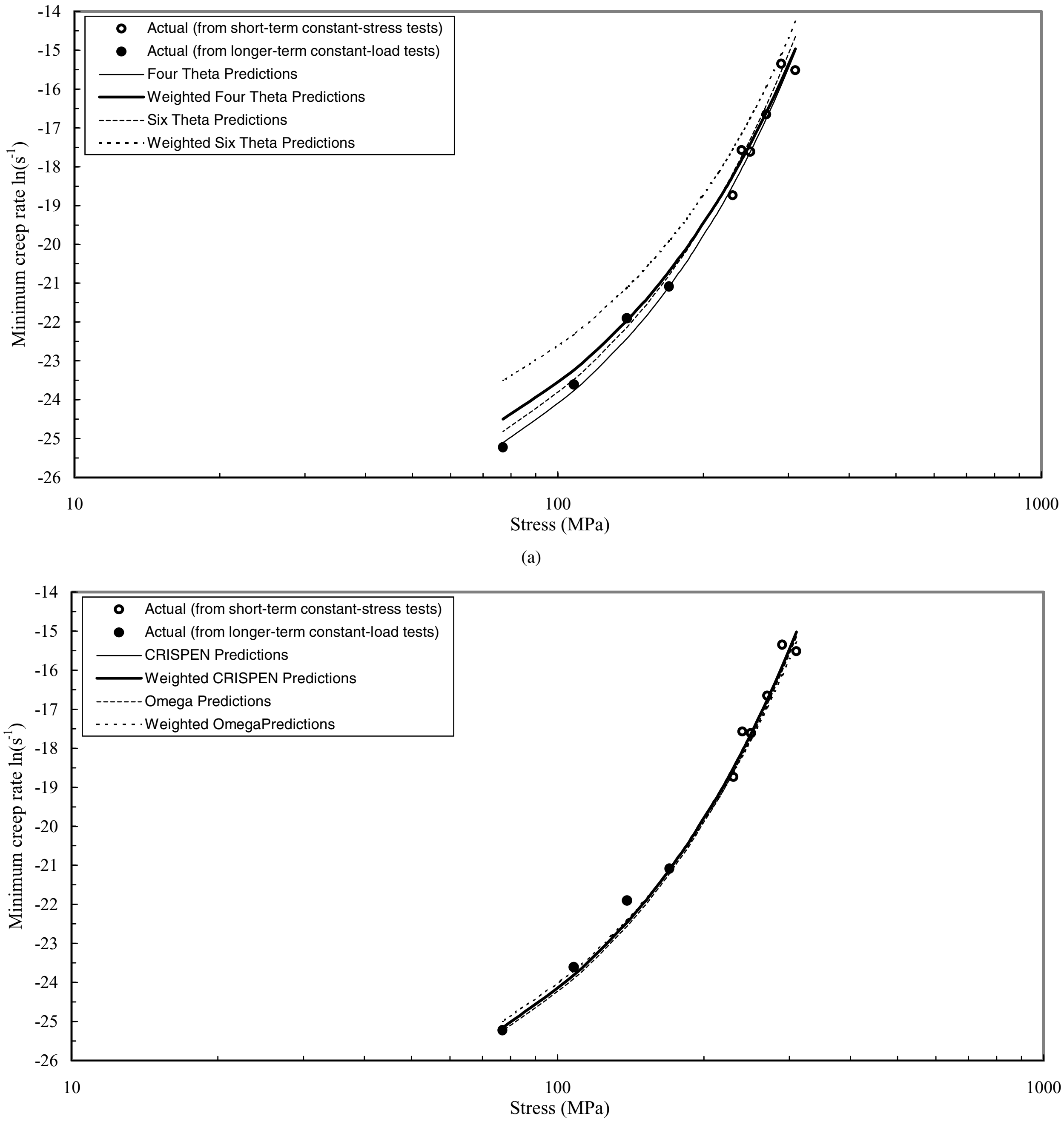



(c)

*Figure 8* (*Continued*).

property predictions using CDM depend on these failure time predictions (Equation 5h shows that time to strain is a function of the time to failure) it is to be expected that these creep property predictions will also be poor and so are not shown in the rest of the paper.

Fig. 8b and c show the predictions made for time to failure using the other prediction techniques, both in weighted and unweighted form. The Omega and CRISPEN predictions are similar irrespective of whether weighting is used. The 6-$\theta$ failure time predictions are always better than their 4-$\Theta$ equivalents and weighting always improves both the 4-$\Theta$ and 6-$\theta$ failure time predictions. The unweighted 4-$\Theta$ and 6-$\theta$ failure time predictions tend to converge at the lower stresses. Of all the prediction techniques, the weighted 6-$\theta$ approach is best, with the weighted 4-$\Theta$ and weighted CRISPEN techniques giving very similar failure time predictions—with 4-$\Theta$ eing better at the lowest stresses. In all case there is a tendency to over predict the actual data obtained from the longer-term tests. This is to be expected because these longer-term tests were done at constant load, whilst the predictions were made from constant stress tests.




(b)

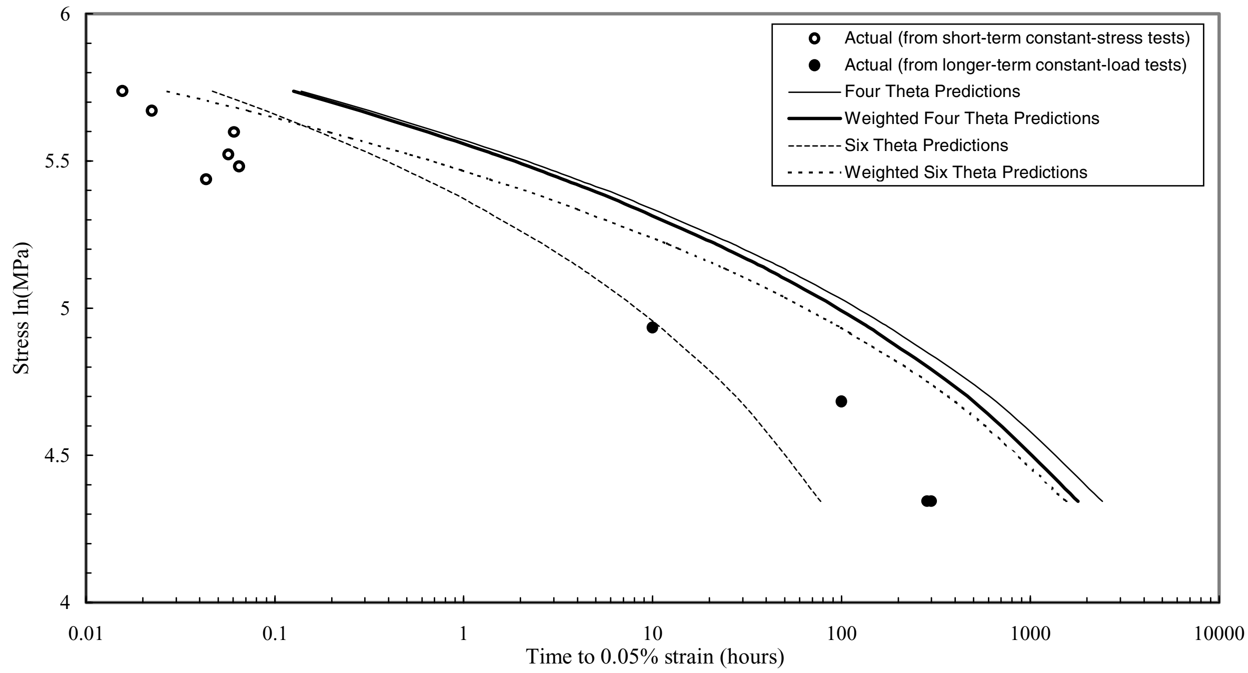
*Figure 9* (a) Constant stress $\ln \tau / \ln \varepsilon_M$ relationships predicted from various lifeing procedures. The plot includes the measured $\varepsilon_M$ values from the short-term constant-stress tests at 823 K and the longer-term results from the constant load tests at 823 K. (b) Constant stress $\ln \tau / \ln \varepsilon_M$ relationships predicted from various lifeing procedures. The plot includes the measured $\varepsilon_M$ values from the short-term constant-stress tests at 823 K and the longer-term results from the constant load tests at 823 K.

### *6.3.2. Minimum creep rates*

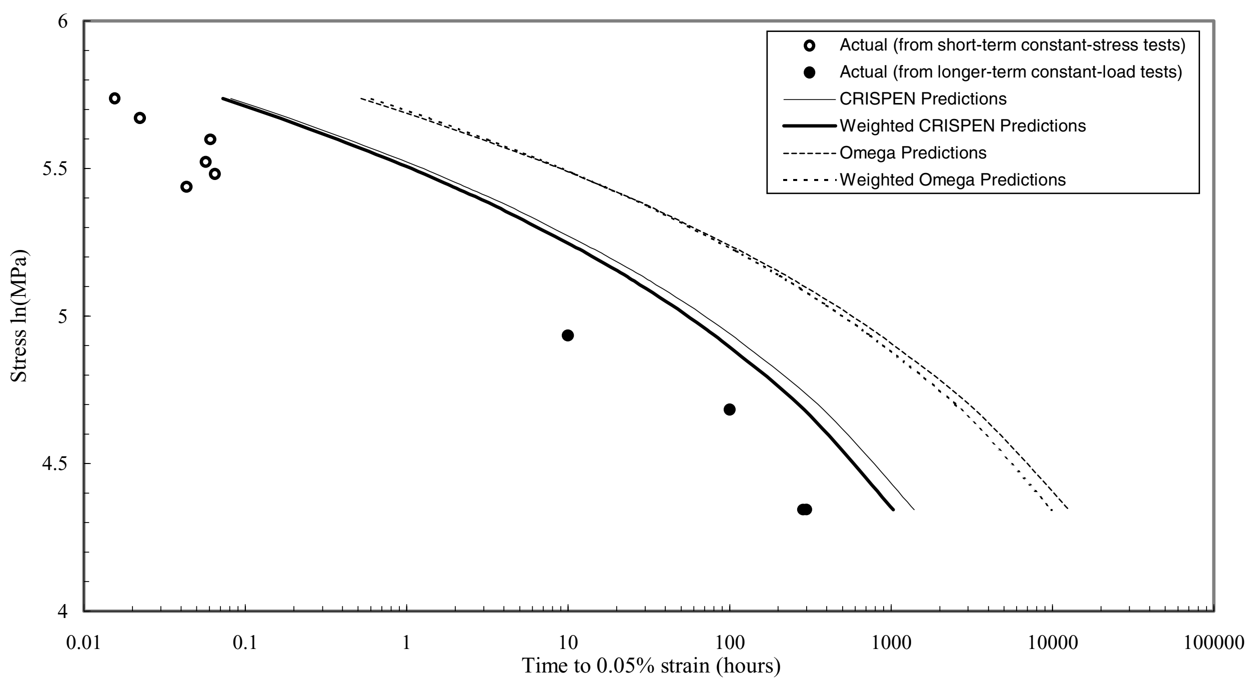
Fig. 9a and b show the predictions made for minimum creep rates using all the prediction techniques (excluding CDM), both in weighted and unweighted form. Again, the Omega and CRISPEN predictions are similar irrespective of whether weighting is used. At the lowest stresses, the 4-$\Theta$ predictions are always better than their 6-$\theta$ equivalents and in this case weighting always leads to a deterioration in both the 4-$\Theta$ and 6-$\theta$ min creep rate predictions. However, in unweighted form, the 4-$\Theta$, CRISPEN and Omega techniques yield equally good long-term minimum creep rate predictions.

### *6.3.3. Times to various strains*

Fig. 10a and b show the predictions made for times to 0.05% strain using all the prediction techniques (excluding CDM), both in weighted and unweighted form. The Omega predictions are by far the worst (both weighted and unweighted) failing even to predict the correct times within the short-term accelerated test data. This is unsurprising given that Equation 4a does not contain a primary component. The 6-$\theta$ technique predicts well the times to 0.05% strain within the short term data and if weighting is used this technique also yields excellent predictions for such times at the middle to lower stresses as well. As expected




(a)

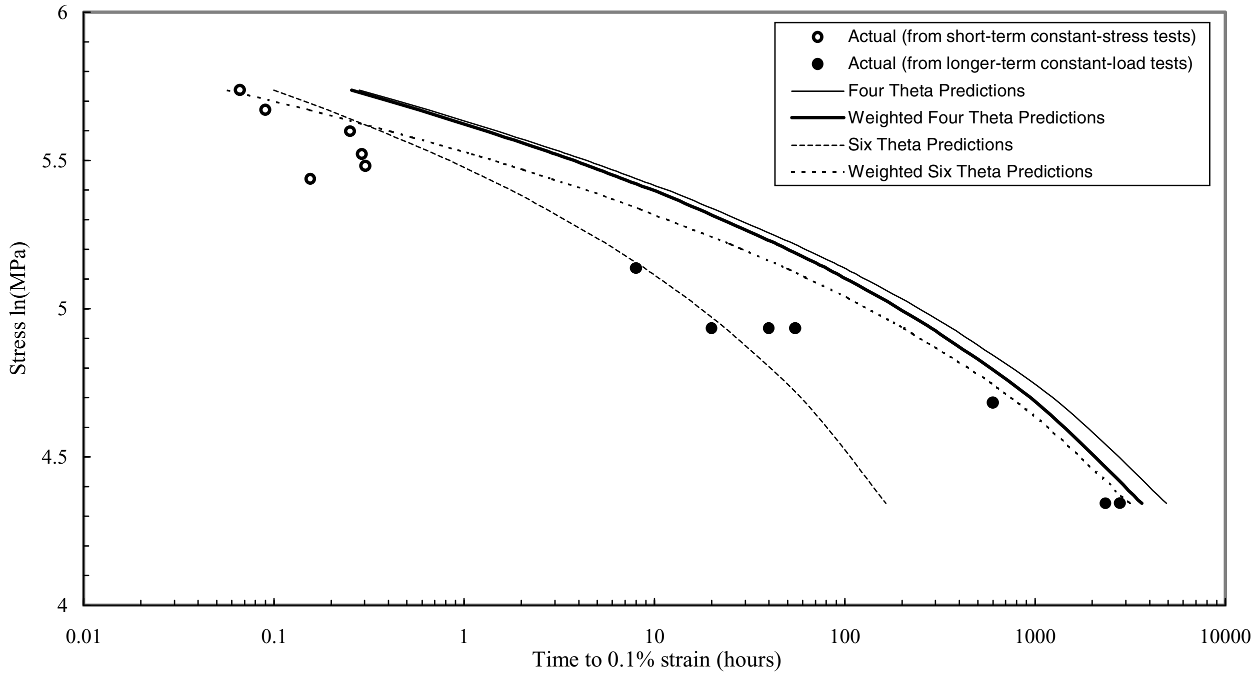



(b)

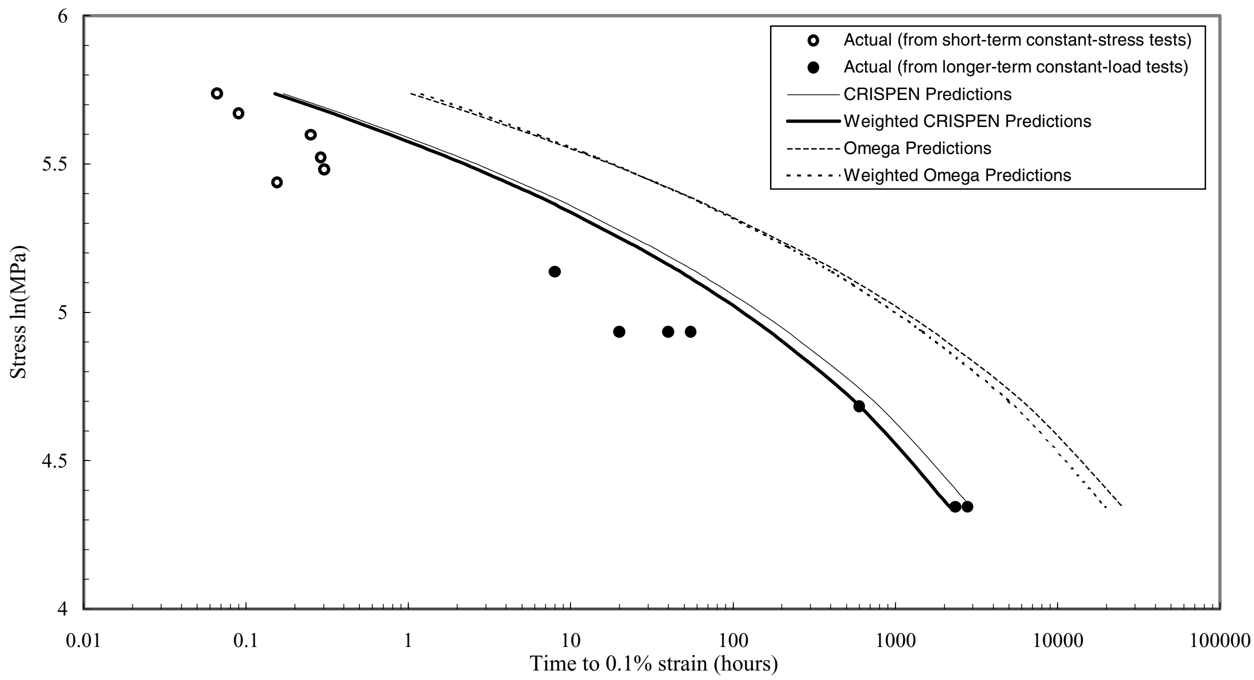
*Figure 10* (a) Predicted $\ln \tau / \ln t_{0.05\%}$ relationships obtained from various lifeing procedures. The plot includes the measured $t_{0.05\%}$ values from the short-term constant-stress tests at 823 K and the longer-term results for the constant load tests at 823 K. (b) Predicted $\ln \tau / \ln t_{0.05\%}$ relationships obtained from various lifeing procedures. The plot includes the measured $t_{0.05\%}$ values from the short-term constant-stress tests at 823 K and the longer-term results for the constant load tests at 823 K.

the 4-Θ technique produces poor predictions of times to 0.05% strain compared to the 6-$\theta$ technique at the high and medium stresses shown in Fig. 10a. Curiously, the predictions from this technique are quite good at the very lowest stresses. The weighted CRISPEN technique produces predictions similar to those obtained using the weighted 6-$\theta$ approach at the middle stresses shown in Fig. 10, but slightly poorer predictions at the very highest and lowest stresses. The weighted 6-$\theta$ predictions appear to fit all the data points shown in Fig. 10 better than any of the other predictions.

As similar picture emerges in Fig. 11a and b. These figures show the predictions made for times to 0.1% strain using all the prediction techniques (excluding CDM), both in weighted and unweighted form. The Omega predictions (weighted or unweighted) are poor at all the stresses. The CRISPEN predictions (weighted and unweighted) are good at high to medium stresses but tend to underestimate at the




(a)




(b)

*Figure 11* (a) Predicted $\ln \tau / \ln t_{0.1\%}$ relationships obtained from various lifeing procedures. The plot includes the measured $t_{0.1\%}$ values from the short-term constant-stress tests at 823 K and the longer-term results for the constant load tests at 823 K. (b) Predicted $\ln \tau / \ln t_{0.1\%}$ relationships obtained from various lifeing procedures. The plot includes the measured $t_{0.1\%}$ values from the short-term constant-stress tests at 823 K and the longer-term results for the constant load tests at 823 K.

lower stresses. The same is true for the weighted 6-$\theta$ technique.

## 7. Conclusions

Of all the prediction techniques studied above, the CDM approach yielded the worst creep property predictions, with predicted times to failure being wholly inadequate. The 6-$\theta$ approach was best for predicted long-term times to failure, followed by the weighted 4-$\Theta$ and then the CRISPEN approaches. The best minimum creep rate predictions came from using the unweighted 4-$\Theta$, or CRISPEN or the Omega techniques (either weighted or unweighted). The 6-$\theta$ approach only yielded adequate minimum creep rate predictions in unweighted form. Finally, times to low strains were best predicted using the 6-$\theta$ and CRISPEN techniques (weighted). The 6-$\theta$ technique was best for interpolating through the short-term data on times to low strains, whilst both techniques yielded very similar predictions at the lowest stresses.

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